Introduction to QCD

7. Exercise

Exercise 1: Dipole Formfactor

The charge distribution and the corresponding form factor in momentum space are related via a Fourier transformation:

$$F(\vec{q}^2) = \int d^3\vec{r}\; e^{-i\vec{q}\vec{r}} \rho(\vec{r})$$

The rms radius of the charge distribution is defined as

$$\langle r^2 \rangle = \frac{\int d^3\vec{r}\; r^2 \rho(\vec{r})}{\int d^3\vec{r}\; \rho(\vec{r})}$$

Show that for any form factor of a rotationally symmetric charge distribution for small q^2

$$F(\vec{q}^2) = F(0)\left[1 - \frac{\vec{q}^2 \langle r^2 \rangle}{6}\right] + \mathcal{O}(q^4)$$

Calculate the charge distribution and the rms radius corresponding to the elastic dipole form factor of the proton

$$F(\vec{q}^2) = \frac{1}{\left(1 + \frac{\vec{q}^2}{M^2}\right)^2}$$

Exercise 2: Scattering off a neutral atom

The scattering of (spinless) electrons off a target with arbitrary charge distribution $\rho(\vec{r})$ can be expressed in the Breit system $[q = (0, \vec{q})]$ by the point-like Rutherford cross section σ_{Ruth} as

$$\frac{d\sigma}{d\Omega} = |F(\vec{q}^2)|^2\, \frac{d\sigma_{Ruth}}{d\Omega}$$

where $F(\vec{q}^2)$ denotes the Fourier-transform of the charge distribution $\rho(\vec{r})$. Calculate the differential cross section for a neutral atom with the charge distribution

$$\rho = \delta_3(\vec{r}) - c\;\delta_1(r - r_0)$$

and determine c such that the atom is neutral.