Introduction to QCD

6. Exercise

Exercise 1: Tunnel probability

Consider the transition probability of a particle in a potential $V(x)$:

$$\langle t_1, x_1 | t_0, x_0 \rangle \sim \int \mathcal{D}x(t) \ \exp\left\{ i \int_{t_0}^{t_1} dt \left[\frac{m}{2} \dot{x}^2 - V(x) \right] \right\}$$

Show by means of the saddle point expansion about a classical path with energy E that

$$\langle t_1, x_1 | t_0, x_0 \rangle \propto \exp\left\{ i \int_{x_0}^{x_1} dx \sqrt{2m[E - V(x)]} \right\}$$

Motivate for the classically forbidden case $E < V(x)$ the expression of the tunnel probability.

Exercise 2: Deep inelastic lepton-nucleon scattering

Show that the hadron tensor can be expressed as the matrix element of the current commutator:

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{spins} \int d^4x \ e^{-iqx} \ \langle \mathcal{N}_p | \left[j_\mu^{elm}(0), j_\nu^{elm}(x) \right] | \mathcal{N}_p \rangle$$