Introduction to QCD

4. Exercise

Exercise 1: Baker–Hausdorff Formula

Derive the special Baker–Hausdorff formula

$$e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]} \quad ; \quad [A,B] \in \mathbb{C}$$

Exercise 2: $\pi^0 \to \gamma\gamma$

The Lorentz structure of the $\pi^0 \to \gamma\gamma$ decay amplitude is dermined uniquely by symmetries and particle properties:

$$\begin{aligned} \mathcal{M}(\pi^0 \to \gamma\gamma) &= i\epsilon^*_\mu(k_1)\epsilon^*_\nu(k_2)T^{\mu\nu} \\ T^{\mu\nu} &= \epsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}T(p^2 = m_\pi^2) \end{aligned}$$

The form factor $T(p^2 = m_\pi^2)$ emerges from analyticity and its asymptotic behaviour as

$$T(p^2 = m_\pi^2) = \frac{\sqrt{2}\alpha}{\pi f_\pi}\left(Q_u^2 - Q_d^2\right)N_c$$

where $f_\pi = (130 \pm 5)$ MeV is the pion decay constant, $\alpha = 1/137.03599911(46)$ the fine structure constant and $Q_u = \frac{2}{3}, Q_d = -\frac{1}{3}$ the quark charges as well as $N_c = 3$ the number of colors. Calculate the decay width and the life time of the photonic pion decay.