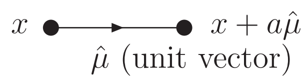
Introduction to QCD

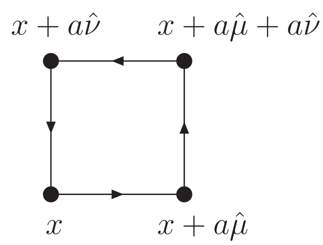
11. Exercise

Exercise 1: Lattice Action

Link variable:
$$U_\mu(x) \equiv U(x + a\hat{\mu}, x) = e^{-aG_\mu(x)}$$
$$= U^{-1}(x, x + a\hat{\mu})$$



Plaquette variable: $U_{\mu\nu}(x)$



Show that Wilson's lattice action

$$S = \frac{6}{g_s^2} \sum_x \sum_{1 \le \mu < \nu \le 4} \left\{ 1 - \frac{1}{6} Tr[U_{\mu\nu}(x) + U_{\mu\nu}^{-1}(x)] \right\}$$

transforms in the continuum limit into the Euklidian action

$$S = -\frac{1}{2g_s^2} \int d^4x \; Tr[G_{\mu\nu}(x) G^{\mu\nu}(x)]$$

with

$$G_{\mu\nu} = \partial_\nu G_\mu - \partial_\mu G_\nu - i[G_\mu, G_\nu]$$

Exercise 2: Richardson-Potential

The Richardson-Potential is defined as:

$$V(r) = -\frac{4}{3} \, \frac{48\pi^2}{33 - 2N_F} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{e^{i\vec{q}\vec{r}}}{\vec{q}^2 \log\left[1 + \frac{\vec{q}^2}{\Lambda^2}\right]}$$

Derive the behaviour of this potential at small and large distances.