Beh.: $[A, B^n] = n[A,B]B^{n-1}$ wenn $[A,B] \in \mathbb{C}$

Bew.: $n=1$: $[A,B] = [A,B]$

$n=2$: $[A,B^2] = AB^2 - B^2A = [A,B]B + B[A,B] = 2[A,B]B$

$$[A,B^{n+1}] = AB^{n+1} - B^{n+1}A = [A,B]B^n + B[A,B^n]$$

$$= [A,B]B^n + n[A,B]B^n$$

$$= (n+1)[A,B]B^n \qquad \text{q.e.d.}$$

$$e^B A e^{-B} = \sum_{i=0}^{\infty} \frac{B^i A}{i!} e^{-B}$$

$$= \sum_{i=0}^{\infty} \frac{[B^i,A] + AB^i}{i!} e^{-B}$$

$$= \sum_{i=1}^{\infty} \frac{-i[A,B]B^{i-1}}{i!} e^{-B} + A \underbrace{e^B e^{-B}}_{\mathbb{1}}$$

$$= -[A,B]e^B e^{-B} + A$$

$$\boxed{e^B A e^{-B} = A + [B,A]} \qquad \boxed{[e^B, A] = [B,A]e^B}$$

$$F(\alpha) = e^{\alpha(A+B)}$$

$$F(\alpha) A F^{-1}(\alpha) = A + \alpha [B, A]$$

$$\Longleftrightarrow F(\alpha) A = A F(\alpha) - \alpha [A, B] F(\alpha)$$

$$\frac{dF(\alpha)}{d\alpha} = F(\alpha)(A+B) = F(\alpha) A + F(\alpha) B$$

$$= A F(\alpha) + F(\alpha) B - \alpha [A, B] F(\alpha)$$

$$\Longrightarrow F(\alpha) = e^{\alpha A} e^{\alpha B} e^{-\frac{\alpha^2}{2}[A, B]}$$

$$\alpha = 1: \boxed{e^{A+B} = e^{A} e^{B} e^{-\frac{1}{2}[A, B]}}$$

$$\sum|M|^2 = \underbrace{\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{\mu\nu\alpha'\beta'}}_{=-2\left(\delta_\alpha^{\alpha'}\delta_\beta^{\beta'}-\delta_\alpha^{\beta'}\delta_\beta^{\alpha'}\right)} k_1^\alpha k_2^\beta k_{1\alpha'} k_{2\beta'}\, |T|^2$$

$$= 2(k_1 k_2)^2 |T|^2 = \frac{m_\pi^4}{2}|T|^2$$

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{1}{2m_\pi}\sum|M|^2 \frac{1}{8\pi}\,\frac{1}{2} = \frac{m_\pi^4}{64\pi m_\pi}|T|^2 = \frac{m_\pi^3}{64\pi}|T|^2$$

$$T = \frac{\sqrt{2}\,\alpha}{\pi f_\pi}\left(Q_u^2 - Q_d^2\right) N_c$$

$$\boxed{\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 m_\pi^3}{32\pi^3 f_\pi^2}\left(Q_u^2 - Q_d^2\right)^2 N_c^2}$$

$N_c = 1$: $\Gamma(\pi^0 \to \gamma\gamma) = (0.868 \pm 0.065)\,\text{eV}$

$N_c = 3$: $\Gamma(\pi^0 \to \gamma\gamma) = (7.81 \pm 0.60)\,\text{eV}$

$m_{\pi^0} = (134.9766 \pm 0.0006)\,\text{MeV}$

$f_\pi = (130 \pm 5)\,\text{MeV}$

$\alpha = 1/137.03599911(46)$

$Q_u = 2/3$

$Q_d = -1/3$