Introduction to QCD

6. Exercise

Exercise 1: Plus Distribution

Show that the following identities are valid:

$$\begin{aligned}
P_{qq}(z) &= C_F \left(\frac{1+z^2}{1-z}\right)_+ \\
&= C_F \left\{2\left(\frac{1}{1-z}\right)_+ - 1 - z + \frac{3}{2}\delta(1-z)\right\} \\
f_{qq}^{DIS}(z) &= C_F \left\{-\frac{1+z^2}{1-z}\log z + (1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ \right. \\
&\quad \left. -\frac{3}{2}\left(\frac{1}{1-z}\right)_+ + 3 + 2z - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-z)\right\} \\
&= C_F \left\{\frac{1+z^2}{1-z}\left[\log\left(\frac{1-z}{z}\right) - \frac{3}{4}\right] + \frac{9}{4} + \frac{5}{4}z\right\}_+
\end{aligned}$$

Exercise 2: Tensor Decomposition

The scalar 1-loop-integrals are defined in general as

$$\begin{aligned}
A_0(m) &= \mu^{2\epsilon}\int \frac{d^n k}{(2\pi)^n}\frac{1}{k^2-m^2} \\
B_0(p;m_0,m_1) &= \mu^{2\epsilon}\int \frac{d^n k}{(2\pi)^n}\frac{1}{(k^2-m_0^2)[(k+p)^2-m_1^2]} \\
C_0(p_1,p_2;m_0,m_1,m_2) &= \mu^{2\epsilon}\int \frac{d^n k}{(2\pi)^n}\frac{1}{(k^2-m_0^2)[(k+p_1)^2-m_1^2][(k+p_2)^2-m_2^2]}
\end{aligned}$$

Consider the following 1-loop 3-point-function:

$$C_\mu(p_1,p_2;m_0,m_1,m_2) = \mu^{2\epsilon}\int \frac{d^n k}{(2\pi)^n}\frac{k_\mu}{(k^2-m_0^2)[(k+p_1)^2-m_1^2][(k+p_2)^2-m_2^2]}$$

Why does this integral follow the decomposition

$$C_\mu(p_1,p_2;m_0,m_1,m_2) = C_1 p_{1\mu} + C_2 p_{2\mu}\ ?$$

Calculate the coefficients $C_{1,2}$ in terms of the scalar integrals B_0, C_0.