Introduction to QCD

5. Exercise

Exercise 1: Moments

Calculate the Mellin moments of all DGLAP splitting functions $P_{ij}(x)$ in lowest order:

$$\begin{aligned}
P_{qq}(x) &= \frac{4}{3}\left(\frac{1+x^2}{1-x}\right)_+ \\
P_{qg}(x) &= \frac{1}{2}\left[x^2+(1-x)^2\right] \\
P_{gq}(x) &= \frac{4}{3}\frac{1+(1-x)^2}{x} \\
P_{gg}(x) &= 6\left\{\left(\frac{1}{1-x}\right)_+ + \frac{1-x}{x} - 1 + x(1-x)\right\} + \frac{33-2N_F}{6}\delta(1-x)
\end{aligned}$$

Exercise 2: Mellin Inversion

The Mellin transform of a function $f(x)$ is defined as

$$F(N) = \int_0^1 dx\; x^{N-1} f(x)$$

Show that this transformation can be inverted (Mellin inversion) in the following way

$$f(x) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} dN\; x^{-N}\; F(N)$$

where σ has to be chosen such that the real parts of all singularities of $F(N)$ are smaller than σ. Moreover, show that the Mellin transform of a convolution of two functions f, g

$$(f\otimes g)(x) = \int_x^1 \frac{dz}{z} f(z) g\left(\frac{x}{z}\right)$$

is the product of both Mellin transforms:

$$\int_0^1 dx\; x^{N-1}(f\otimes g)(x) = F(N)G(N)$$