Introduction to QCD

1. Exercise

Exercise 1: SU(3)–Invariance

The objects $a_i^{(k)}, b_i^{(k)}$ $(i,k = 1,2,3)$ are vectors that transform under SU(3) as

$$\begin{aligned} a_i^{\prime(k)} &= U_{ij} a_j^{(k)} \\ b_i^{\prime(k)} &= U_{ij}^* b_j^{(k)} \end{aligned}$$

where $U \in \mathrm{SU}(3)$. Show that

$$b_i^{(1)} \delta_{ij} a_j^{(1)} \qquad , \qquad \epsilon_{ijk} a_i^{(1)} a_j^{(2)} a_k^{(3)} \qquad , \qquad \epsilon_{ijk} b_i^{(1)} b_j^{(2)} b_k^{(3)}$$

are invariant under SU(3) transformations.

Exercise 2: Dispersion Relations

Consider a scattering problem in optics:
Incoming (planar) wave in one space dimension:

$$A_{in}(t,x) = \int_{-\infty}^{\infty} d\omega \; a(\omega) \; e^{-i\omega(t-x)} \qquad \text{with} \qquad A_{in}(t,x) = 0 \quad \text{for} \quad x > t$$

Scattered wave:

$$A_{out}(t,x) = \frac{1}{x} \int_{-\infty}^{\infty} d\omega \; f(\omega) \; a(\omega) \; e^{-i\omega(t-x)}$$

Show by means of causality that $f(\omega)$ can be continued analytically in the upper complex half plane $\Im m\omega \geq 0$. If the wave functions A_{in}, A_{out} are real, they can also be continued analytically into the lower complex halfplane due to $a_{in}(-\omega) = a_{in}^*(\omega)$. Derive under these conditions by means of Cauchy's integral and the distribution relation

$$\frac{1}{z \pm i\epsilon} = P\left(\frac{1}{z}\right) \mp i\pi\delta(z)$$

(P is Cauchy's principle value integration) the dispersion relation:

$$f(\omega) = f(\omega + i\epsilon) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\Im m f(\omega')}{\omega' - \omega - i\epsilon}$$

if $f(\omega) \to 0$ for $|\omega| \to \infty$, and

$$\frac{f(\omega)}{\omega} = \frac{f(0)}{\omega} + \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\Im m f(\omega')}{(\omega' - i\epsilon)(\omega' - \omega - i\epsilon)}$$

if $f(\omega)/\omega \to 0$ for $|\omega| \to \infty$.