) $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}^2 + \bar{q}(i\not{D} - m_q)q - \frac{1}{\xi}\mathrm{Tr}(\partial G)^2$

$$i D_\mu = i\partial_\mu - g_s G_\mu^a T^a$$

$$G_{\mu\nu} = \partial_\nu G_\mu - \partial_\mu G_\nu - i g_s [G_\mu, G_\nu]$$

(i) <u>Gluon-Propagator:</u> bilinearer Term in G

$$\mathcal{L}_{GG} = -\frac{1}{4}(\partial_\nu G_\mu^a - \partial_\mu G_\nu^a)^2 - \frac{1}{2\xi}(\partial^\lambda G_\mu^a)(\partial^\nu G_\nu^a)$$

$$\rightarrow \left\{-\frac{1}{2}g^{\mu\nu}\partial^2 + \frac{1}{2}\partial^\mu\partial^\nu - \frac{1}{2\xi}\partial^\mu\partial^\nu\right\} G_\mu^a G_\nu^b \delta_{ab}$$

$$\Rightarrow D_{\mu\nu}^{-1} = \left\{-g_{\mu\nu}q^2 + \left(1-\frac{1}{\xi}\right)q_\mu q_\nu\right\} \underbrace{\frac{1}{2}\cdot 2!}_{=1}$$

$$D_{\mu\nu} = A\, g_{\mu\nu} + B\, q_\mu q_\nu$$

$$D_\mu{}^\rho D_{\rho\nu}^{-1} \overset{!}{=} g_{\mu\nu} = -A^2 g_{\mu\nu} + \left(1-\frac{1}{\xi}\right)q_\mu q_\nu A - Bq^2 q_\mu q_\nu + Bq^2\left(1-\frac{1}{\xi}\right)q_\mu q_\nu$$

$$= -Aq^2 g_{\mu\nu} + \left[\left(1-\frac{1}{\xi}\right)A - B\frac{q^2}{\xi}\right]q_\mu q_\nu$$

$$\Rightarrow A = -\frac{1}{q^2} \qquad B = \frac{1-\xi}{q^4}$$

$$\Rightarrow D_{\mu\nu} = \frac{-g_{\mu\nu} + (1-\xi)\frac{q_\mu q_\nu}{q^2}}{q^2 + i\varepsilon}$$

 $i D_{\mu\nu}\delta_{ab}$

(ii) <u>3-Gluon-Vertex:</u> trilinearer Term in G

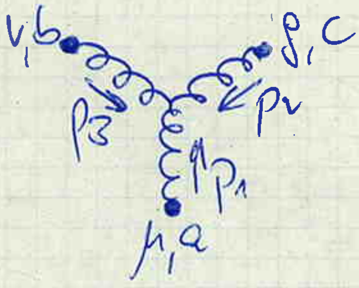
$$\mathcal{L}_{GGG} = \frac{i}{2}g_s\, 2\,\mathrm{Tr}\left\{(\partial_\nu G_\mu - \partial_\mu G_\nu)[G_\mu, G_\nu]\right\}$$

$$= -g_s f_{abc} G_\mu^a G_\nu^b (\partial_\nu G_\mu^d - \partial_\mu G_\nu^d)\,\mathrm{Tr}(T^d T^c)$$

$$= -\frac{g_s}{2} f_{abc} G^a_\mu G^b_\nu (\partial^\nu G^{c\mu} - \partial^\mu G^{c\nu})$$

$$\longrightarrow -\frac{g_s}{2} f_{abc} \underbrace{G^a_\mu G^b_\nu}_{\to 2!} G^c_\rho (-i q_3^\nu g^{\rho\mu} + i q_3^\mu g^{\rho\nu})$$

alle Permutationen:

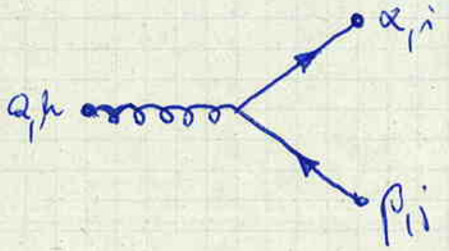


Feynman-Regel: $-i\frac{g_s}{2}\{-ip_3^\nu g^{\rho\mu} + i p_3^\mu g^{\rho\nu} - i p_2^\mu g^{\rho\nu} + i p_2^\rho g^{\mu\nu} - i p_1^\rho g^{\mu\nu} + i p_1^\nu g^{\mu\rho}\} 2! f_{abc}$

$$= -g_s f_{abc}\{(p_1-p_2)^\rho g^{\mu\nu} + (p_3-p_1)^\nu g^{\mu\rho} + (p_2-p_3)^\mu g^{\nu\rho}\}$$

(iii) Quark-Gluon-Vertex:

$$\mathcal{L}_{qqg} = -g_s \bar q \not{G} q = -g_s \bar q_{\alpha,i} \gamma^\mu_{\alpha\beta} q_{\beta,j} T^a_{ij} G^a_\mu$$



$-i g_s \gamma^\mu_{\alpha\beta} T^a_{ij}$

) Ladungsquantisierung

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}^2 + \sum_k \overline{\Psi}_k (i\not{\partial} - Q_k e \not{A} - m_k)\Psi_k$$

$$\Psi_k \rightarrow \Psi'_k = e^{-i\alpha Q_k e}\Psi$$

$$\overline{\Psi}_k \rightarrow \overline{\Psi}'_k = e^{i\alpha Q_k e}\overline{\Psi}_k$$

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \alpha \implies F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu \rightarrow F'_{\mu\nu} = F_{\mu\nu}$$

$$\mathcal{L}_{QED} \rightarrow \mathcal{L}'_{QED} = -\frac{1}{4}F'^2_{\mu\nu} + \sum_k \overline{\Psi}'_k (i\not{\partial} - Q_k e \not{A}' - m_k)\Psi'_k$$

$$= -\frac{1}{4}F_{\mu\nu}^2 + \sum_k \overline{\Psi}_k e^{i\alpha Q_k e} e^{-i\alpha Q_k e}[i\not{\partial} + Q_k e \not{\partial}\alpha$$

$$- Q_k e \not{A} - Q_k e \not{\partial}\alpha - m_k]\Psi_k$$

$$= -\frac{1}{4}F_{\mu\nu}^2 + \sum_k \overline{\Psi}_k (i\not{\partial} - Q_k e \not{A} - m_k)\Psi_k = \mathcal{L}_{QED}$$

Q_k beliebig! $\implies$ <u>keine</u> Ladungsquantisierung

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^{a\,2}_{\mu\nu} + \overline{q}(i\not{\partial} - \lambda_q g_s \not{G} - m_q)q$$

gemäß Vorlesung: $q \rightarrow q' = Sq \qquad S = e^{-i\alpha_k T^k}$

$$\overline{q} \rightarrow \overline{q}' = \overline{q}S^{-1}$$

$$G_\mu \rightarrow SG_\mu S^{-1} - \frac{i}{g_s} S\partial_\mu S^{-1}$$

$$\implies D \rightarrow D' = SDS^{-1}$$

$$G_{\mu\nu} \rightarrow G'_{\mu\nu} = SG_{\mu\nu}S^{-1}$$

$$\mathcal{L}'_{QCD} = -\frac{1}{2}\mathrm{Tr}(G'^2_{\mu\nu}) + \overline{q}'(i\not{\partial} - \lambda_q g_s \not{G}' - m_q)q'$$

$$= -\frac{1}{2}\mathrm{Tr}(G^2_{\mu\nu}) + \overline{q}S^{-1}[i\not{\partial}S + Si\not{\partial} - \lambda_q g_s(S\not{G} - \frac{i}{g_s}S(\not{\partial}S^{-1})S) - m_q S]q$$

$$= -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}^2 + \bar{q}\left[i\not\partial - \lambda_q g_s \not{G} - m_q + i S^{-1}(\not\partial S) + \underbrace{i\lambda_q(\not\partial S^{-1})S}_{=-i\lambda_q S^{-1}(\not\partial S)}\right] q$$

$$= \mathcal{L}_{QCD} + \bar{q}\, i(1-\lambda_q)\, S^{-1}(\not\partial S)\, q$$

$$= \mathcal{L}_{QCD} + i(1-\lambda_q)\, \bar{q}\, S^{-1}(\not\partial S)\, q$$

$\Rightarrow$ invariant <u>nur</u> für $\underline{\underline{\lambda_q = 1}}$

$\Rightarrow$ Ladungsquantisierung in nicht-abel'schen Eichtheorien