Introduction to QCD

12. Exercise

Exercise 1: (Axial-)Vector Current

Calculate the Noether currents and their divergences explicitly for the Lagrangian

$$\mathcal{L} = \sum_{j=1}^{n} \bar{q}_j (i \not{\partial} - m_j) q_j = \bar{q}(i \not{\partial} - M) q$$

with the mass matrix $M_{ij} = m_i \delta_{ij}$ and the transformations

(a) $q' = e^{i\alpha^a T^a} q \quad \rightarrow \quad$ current j^a, charge Q^a

(b) $q' = e^{i\alpha^a T^a \gamma_5} q \quad \rightarrow \quad$ current j_5^a, charge Q_5^a

Exercise 2: Current Algebra

Show that with the following definitions

$$[T^a, T^b] = i f_{abc} T^c \qquad \text{and} \qquad Q_\pm^a = \frac{1}{2}(Q^a \pm Q_5^a),$$

for equal times

$$[Q^a, Q^b] = [Q_5^a, Q_5^b] = i f_{abc} Q^c$$
$$[Q^a, Q_5^b] = \quad i f_{abc} Q_5^c$$

and therefore

$$\begin{aligned} [Q_\pm^a, Q_\pm^b] &= i f_{abc} Q_\pm^c \\ [Q_+^a, Q_-^b] &= 0 \end{aligned}$$

Hint: Use the canonical anticommutation rules for quark fields as well the operator identity

$$[AB, CD] = A\{B, C\}D - C\{A, D\}B + CA\{B, D\} - \{A, C\}BD$$