5. Übung (Musterlösung)

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$$\langle t_1, x_1 | t_0, x_0 \rangle \sim \int \mathcal{D}x(t)\, e^{iS}$$

mit $S = \int_{t_0}^{t_1} dt\, L(\dot{x}, x)$ $\qquad L(\dot{x}, x) = \frac{m}{2}\dot{x}^2 - V(x)$

$$S(x) = S(x_{kl}) + \left[S(x) - S(x_{kl})\right] = S_{kl} + \left[S(x_{kl}+y) - S(x_{kl})\right]$$

Substitution: $y(t) = x(t) - x_{kl}(t) \Longrightarrow \mathcal{D}y(t) = \mathcal{D}x(t)$

$$S(x) = S_{kl}(x_{kl}) + \underbrace{\frac{\partial S}{\partial x_{kl}}}_{=\delta S = 0} y + \frac{\partial^2 S}{\partial x_{kl}^2} y^2 + \dots$$

$$\langle t_1, x_1 | t_0, x_0 \rangle \sim \int \mathcal{D}x(t)\, e^{iS} \approx e^{iS(x_{kl})} \int \mathcal{D}y(t)\, e^{i \frac{\partial^2 S}{\partial x_{kl}^2} y^2}$$

$$\propto e^{iS(x_{kl})}$$

$$\delta S = \int_{t_0}^{t_1} dt \left\{ \frac{\partial L}{\partial \dot{x}} \delta\dot{x} + \frac{\partial L}{\partial x}\delta x \right\} = \int_{t_0}^{t_1} dt \left\{ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\delta x\right) + \left[\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}}\right]\delta x \right\} = 0$$

$$\delta x(t_0) = \delta x(t_1) = 0$$

$$\longrightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \quad \text{für } x_{kl}(t)$$

$$m\ddot{x} = -V'(x) \Longleftrightarrow m\ddot{x}\dot{x} = -\dot{x}V'(x) = -\frac{d}{dt}V(x)$$

$$\Longleftrightarrow \frac{m}{2}\dot{x}^2 = E - V(x) \Longleftrightarrow m\dot{x} = \sqrt{2m(E - V(x))}\,; \quad V(x) = E - \frac{m}{2}\dot{x}^2$$

$$S_{kl} = \int_{t_0}^{t_1} dt \left\{ m\dot{x}^2 - E \right\} = \int_{t_0}^{t_1} dt \left\{ \dot{x}\sqrt{2m(E-V(x))} - E \right\}$$

②

$$S_{kl} = -E(t_1 - t_0) + \int_{x_0}^{x_1} dx \sqrt{2m(E - V(x))}$$

$$\langle t_1 x_1 | t_0 x_0 \rangle \propto e^{iS_{kl}} = e^{-iE(t_1-t_0)} \, e^{i\int_{x_0}^{x_1} dx \sqrt{2m[E-V(x)]}}$$

$$\propto e^{i\int_{x_0}^{x_1} dx \sqrt{2m[E-V(x)]}} \qquad (E \to E + i\varepsilon)$$

$E < V(x)$: $\sqrt{2m(E-V(x))} = +i\sqrt{2m[V(x)-E]}$

$$\langle t_1 x_1 | t_0 x_0 \rangle \propto e^{-\int_{x_0}^{x_1} dx \sqrt{2m[V(x)-E]}}$$

$\Longrightarrow$ exponentielle Dämpfung $\longrightarrow$ <u>Tunneleffekt</u>

② Gemäß Vorlesung:

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{\text{Spins}} \int d^4x \, e^{-iqx} \langle N_p | j_\mu^{elm}(0) \, j_\nu^{elm}(x) | N_p \rangle$$

2. Term des Kommutators:

$$\int d^4x \, e^{-iqx} \langle N_p | j_\nu^{elm}(x) \, j_\mu^{elm}(0) | N_p \rangle$$

$$= \sum_X \int d^4x \, e^{-i(q-p+p_X)} \langle N_p | j_\nu^{elm}(0) | X \rangle \langle X | j_\mu^{elm}(0) | N_p \rangle$$

$$= \sum_X (2\pi)^4 \delta_4(q + p_X - p) \langle N_p | j_\nu^{elm}(0) | X \rangle \langle X | j_\mu^{elm}(0) | N_p \rangle$$

$\Longrightarrow q = p - p_X$

$q^0 = M_p - E_X = E - E' \Longrightarrow E_X = M_p - (E - E') < M_p$

$\Longrightarrow M_X < M_p$ ↯

da Nukleon N_p das <u>leichteste</u> Baryon mit $B=1$ ist.

$\Longrightarrow$ 2. Term $= 0$

$$\Longrightarrow W_{\mu\nu} = \frac{1}{8\pi} \sum_{\text{Spins}} \int d^4x \, e^{-iqx} \langle N_p | [j_\mu^{elm}(0), j_\nu^{elm}(x)] | N_p \rangle$$