Introduction to QCD

3. Exercise

Exercise 1: Free Action Functional

Show that the free bosonic action functional

$$W_0(j) = \langle 0|T \exp\left\{ i \int d^4x \; j(x) \; \phi(x) \right\} |0\rangle$$

can be rewritten as

$$W_0(j) = \exp\left\{ -\frac{i}{2} \int d^4x \; d^4y \; j(x) \; \Delta_F(x-y) \; j(y) \right\}$$

where $i\Delta_F(x-y) = \langle 0|T\{\phi(x)\phi(y)\}|0\rangle$ denotes the Feynman propagator in coordinate space. Here $\phi(x)$ is the (real) bosonic field operator and $j(x)$ the external source. The symbol T denotes the time ordering operator.

Exercise 2: Mathews-Salam Formulae

Prove the following Mathews-Salam formulae for path integrals over Grassmann variables

$$\frac{\int \mathcal{D}\bar{\eta} \; \mathcal{D}\eta \; e^{-\bar{\eta}Q\eta}}{\int \mathcal{D}\bar{\eta} \; \mathcal{D}\eta \; e^{-\bar{\eta}\eta}} = DetQ$$

$$\frac{\int \mathcal{D}\bar{\eta} \; \mathcal{D}\eta \; \eta_i\bar{\eta}_j \; e^{-\bar{\eta}Q\eta}}{\int \mathcal{D}\bar{\eta} \; \mathcal{D}\eta \; e^{-\bar{\eta}\eta}} = Q^{-1}_{ij} DetQ$$

where $\eta, \bar{\eta}$ are multi-component Grassmann variables.