7. Übung (Musterlösung)

1 Plusvorschrift

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{1-z}\right)_+ = C_F\left\{\frac{1+z^2}{1-z} - \delta(1-z)\int_0^1 dz'\,\frac{1+z'^2}{1-z'}\right\}$$

$$P'_{qq}(z) = C_F\left\{2\left(\frac{1}{1-z}\right)_+ - 1 - z + \frac{3}{2}\,\delta(1-z)\right\}$$

$$= C_F\left\{\frac{2}{1-z} - 2\delta(1-z)\int_0^1 dz'\,\frac{1}{1-z'} - 1 - z + \frac{3}{2}\delta(1-z)\right\}$$

$$P_{qq}(z) - P'_{qq}(z) = C_F\left\{\frac{1+z^2}{1-z} - \frac{2}{1-z} + 1 + z + \delta(1-z)\int_0^1 dz'\left[-\frac{3}{2} - \frac{1+z'^2}{1-z'} + \frac{2}{1-z'}\right]\right\}$$

$$= C_F\,\delta(1-z)\left\{-\frac{3}{2} + \int_0^1 dz'\,(1+z')\right\} = 0$$

$$\longrightarrow\ P_{qq}(z) = P'_{qq}(z)$$

$$f_{qq}(z) = C_F\left\{-\frac{1+z^2}{1-z}\log z + (1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{3}{2}\left(\frac{1}{1-z}\right)_+ + 3 + 2z - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-z)\right\}$$

$$= C_F\left\{\frac{1+z^2}{1-z}\left[\log\frac{1-z}{z} - \frac{3}{4}\right] + \frac{9}{4} + \frac{5}{4}z + \delta(1-z)\left[-\frac{9}{2} - \frac{\pi^2}{3} - \int_0^1 dz'\left(2\frac{\log(1-z')}{1-z'} + \frac{3}{2(1-z')}\right)\right]\right\}$$

$$f'_{qq}(z) = C_F\left\{\frac{1+z^2}{1-z}\left[\log\frac{1-z}{z} - \frac{3}{4}\right] + \frac{9}{4} + \frac{5}{4}z\right.$$

$$\left. - \delta(1-z)\int_0^1 dz'\left[\frac{1+z'^2}{1-z'}\left(\log\frac{1-z'}{z'} - \frac{3}{4}\right) + \frac{9}{4} + \frac{5}{4}z'\right]\right\}$$

$$f_{qq}(z) - f'_{qq}(z) = C_F\,\delta(1-z)\left\{-\frac{9}{2} - \frac{\pi^2}{3} + \int_0^1 dz'\left[\frac{1+z'^2-2}{1-z'}\log(1-z') - \frac{1+z'^2}{1-z'}\log z'\right.\right.$$

$$\left.\left. - \frac{3}{4}\,\frac{1+z'^2-2}{1-z'} + \frac{9}{4} + \frac{5}{4}z'\right]\right\}$$

$$= C_F\,\delta(1-z)\left\{-\frac{9}{2} - \frac{\pi^2}{3} + \frac{9}{4} + \frac{5}{8} + \frac{3}{4} + \frac{3}{8} + \int_0^1 dz'\left[-(2-z')\log z' - \frac{2-2z'+z'^2}{z'}\log(1-z')\right]\right.$$

$$= C_F \delta(1-z)\Big\{-\frac{1}{2} - \frac{\pi^2}{3} + \Big[2 - \frac{1}{4} + \frac{\pi^2}{3} - 2 + \frac{3}{4}\Big]\Big\} = 0$$

$$\Longrightarrow f_{qq}(z) = f'_{qq}(z)$$

Aufgabe 2:

$$C_0(p_1, p_2; m_0, m_1, m_2) = F(p_1^2, p_2^2, p_1 p_2; m_0, m_1, m_2)$$

$C_\mu(p_1, p_2; m_0, m_1, m_2)$ Vierervektor

einzige Vierervektoren: p_1^μ, p_2^μ

$$\Longrightarrow C^\mu = C_1 p_1^\mu + C_2 p_2^\mu$$

$$p_1^\mu C_\mu = p_1^2 C_1 + (p_1 p_2) C_2$$

$$p_2^\mu C_\mu = (p_1 p_2) C_1 + p_2^2 C_2$$

$$\Longrightarrow \boxed{C_1 = \frac{p_2^2\, p_1^\mu C_\mu - (p_1 p_2)\, p_2^\mu C_\mu}{p_1^2 p_2^2 - (p_1 p_2)^2} \qquad C_2 = \frac{p_1^2\, p_2^\mu C_\mu - (p_1 p_2)\, p_1^\mu C_\mu}{p_1^2 p_2^2 - (p_1 p_2)^2}}$$

$$k p_i = \frac{1}{2}\Big\{\big[(k + p_i)^2 - m_i^2\big] - (k^2 - m_0^2) - (p_i^2 + m_0^2 - m_i^2)\Big\}$$

$$\longrightarrow \boxed{p_i^\mu C_\mu = \frac{1}{2}\Big\{ \underset{(j \neq i)}{B_0(p_j; m_0, m_1)} - B_0(p_1 - p_2; m_1, m_2) - (p_i^2 + m_0^2 - m_i^2)\, C_0(p_1, p_2; m_0, m_1, m_2)}$$