HDECAY: Twenty++ Years After

Abdelhak Djouadi\textsuperscript{a}, Jan Kalinowski\textsuperscript{b,c,}\textsuperscript{*}, Margarete M"uhleitner\textsuperscript{d}, and Michael Spira\textsuperscript{e}

\textsuperscript{a}Laboratoire de Physique Théorique, CNRS – UMR 8627, Université Paris-Sud XI, F-91405 Orsay Cedex, France
\textsuperscript{b}Faculty of Physics, University of Warsaw, ul. Pasteura 5, PL-02-093 Warsaw, Poland
\textsuperscript{c}CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland
\textsuperscript{d}Institute for Theoretical Physics, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
\textsuperscript{e}Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland

Abstract

The program \textsc{HDECAY} determines the partial decay widths and branching ratios of the Higgs bosons within the Standard Model with three and four generations of fermions, including the case when the Higgs couplings are rescaled, a general two–Higgs doublet model where the Higgs sector is extended and incorporates five physical states and its most studied incarnation, the minimal supersymmetric Standard Model (MSSM). The program addresses all decay channels including the dominant higher-order effects such as radiative corrections and multi-body channels. Since the first launch of the program, more than twenty years ago, important aspects and new ingredients have been incorporated. In this update of the program description, some of the developments are summarized while others are discussed in some detail.

\textit{Key words:} Higgs boson; decay widths; decay branching ratios; Standard Model; two-Higgs doublets; supersymmetric extensions; higher orders.

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\textsuperscript{*}Corresponding author. \textit{E-mail address:} Jan.Kalinowski@fuw.edu.pl

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1. Introduction

The FORTRAN code HDECAY [1], released more than twenty years ago (on arXiv in April 1997)\(^1\), addresses a crucial issue in the phenomenology of the Higgs particles which, in the context of the Standard Model (SM) of particle physics, have been predicted long ago [4] and were only discovered in 2012 at the CERN Large Hadron Collider (LHC) [5]. Indeed, the strategies for Higgs bosons searches at high-energy colliders, such as the LHC, exploit various Higgs decay channels. The detection strategies depend not only on the experimental setup (for instance hadron versus lepton colliders) but also on the theoretical scenarios: the SM or some of its extensions, such as the two-Higgs doublet model (2HDM), or the Minimal Supersymmetric Standard Model (MSSM), or variants such as including a fourth generation of fermions, or Higgs particles with rescaled couplings to fermions and gauge bosons.

In the SM, the electroweak symmetry is hidden by one doublet scalar field leading to the existence of one single neutral Higgs boson, denoted as \(H\) [4]. The Higgs boson couplings to the 3-generation fermions and to gauge bosons are related to the masses of these particles and are thus determined by the symmetry breaking mechanism. In contrast, the mass \(M_H\) of the Higgs boson itself is undetermined by the model and is known to have the value of \(M_H \approx 125\) GeV only after it was observed by the ATLAS and CMS collaborations [5]. Since this mass value is known, all Higgs couplings, including the self-couplings, are fixed and the properties of the \(H\) boson, production cross sections and partial decay widths, are uniquely determined.

The situation is more complicated in the beyond the SM context and,

\(^1\)Actually, we are not exactly "twenty years after" [2] the first release of the program but closer to 42/2 years and thus, maybe half-way in our search of the answer to everything in the universe [3].
for instance, additional neutral and charged Higgs bosons are predicted in 2HDMs [6, 7], as realized in the MSSM [6, 8, 9]. In these models, there are altogether five physical Higgs bosons: 2 CP–even Higgs bosons $h$ and $H$, with $M_h \leq M_H$, a CP–odd or pseudoscalar $A$ and two charged $H^\pm$ bosons. Either the lighter or the heavier CP–even Higgs boson can be identified with the one already observed. The four masses $M_h, M_A, M_H$ and $M_{H^\pm}$, as well as the ratio of the two Higgs field vacuum expectation values $\tan \beta = v_2/v_1$ and the mixing angle $\alpha$ that diagonalises the two CP–even Higgs states, are unrelated in a general 2HDM. In the MSSM, however, supersymmetry imposes strong constraints on the parameters and, in fact, only two of them, usually taken to be $\tan \beta$ and $M_A$, are independent at tree level.

The MSSM at larger $A$ masses approaches the decoupling regime [10] in which the lighter CP–even $h$ state will have almost SM–like couplings while the four states $H, A$ and $H^\pm$ become heavy, degenerate in mass and decouple from the massive gauge bosons. In the 2HDM, to cope naturally with the fact that the observed Higgs boson is SM–like, one invokes the so–called alignment limit [11] in which only one Higgs doublet gives masses to the $V = W/Z$ bosons. In this case, the mixing angle $\alpha$ is such that the Higgs couplings of one of the CP–even Higgs bosons are the same as the ones of the SM Higgs state. In this case too, the second CP–even state will no longer couple to massive gauge bosons as also does the pseudoscalar $A$ in general.

It was, and still is, of vital importance to have reliable predictions for the branching ratios of the Higgs boson decays for these theoretical models. The program HDECAY calculates the Higgs boson partial decay widths and the decay branching ratios within the SM, 2HDM and MSSM scenarios\(^2\). In its first version, the main features of the program were as follows:

- Included are all decay channels that are kinematically allowed and which have branching ratios larger than $10^{-4}$, i.e. the loop mediated, the three-body decay modes and in the MSSM the cascade and the supersymmetric decay channels [9, 18, 19].

\(^2\)The program has been adapted to very special cases and some versions exist to deal with specific situations like the MSSM with and without boundary conditions at the high-energy scale, the SUSY–HIT program [12], the extension to the Next-to-Minimal Supersymmetric SM in NMSSMCalc [13], the implementation of the Higgs effective Lagrangian with a linear or non-linear realization of electroweak symmetry breaking, eHDECAY [14], the implementation of the singlet extended SM or 2HDM in sHDECAY [15] and N2HDECAY [16], respectively, and the version for the CP-violating 2HDM C2HDM-HDECAY [17].
– All relevant higher-order QCD corrections to the decays into quark pairs and to the loop mediated decays into gluons are incorporated [18, 20].

– Double off-shell decays of the CP-even Higgs bosons into massive gauge bosons which then decay into four massless fermions, and all important below-threshold three- and four-body decays are included [21, 22].

– In the MSSM, the radiative corrections in the effective potential approach with full mixing in the stop/sbottom sectors are incorporated using the renormalization group improved values of the Higgs masses and couplings thus including the relevant next-to-leading-order (NLO) and next-to-NLO (NNLO) corrections [23, 24, 25].

– In the MSSM, all the decays into supersymmetric (SUSY) particles (neutralinos, charginos, sleptons and squarks including mixing in the stop, sbottom and stau sectors) when kinematically allowed are calculated [26]. The SUSY particles are also included in the loop mediated $\gamma\gamma, Z\gamma$ and $gg$ decay channels.

The program, written in FORTRAN77, provided a very flexible and convenient use, fitting to all options of phenomenological relevance. The basic input parameters, fermion and gauge boson masses and their total widths, coupling constants and, in the MSSM, soft SUSY-breaking parameters can be chosen from an input file `hdecay.in`. In this file several flags allow switching on/off or changing some options [e.g. choosing a particular Higgs boson, including/excluding the multi-body or SUSY decays, or including/excluding some specific higher-order QCD corrections].

All the relevant information is given in the original publication [1] to which we refer for details. However, since the first release of the original version of the program some bugs have been fixed, a number of improvements and new theoretical calculations have been implemented. Earlier important modifications were documented, besides those of Refs. [12, 14] where the special extensions SUSY-HIT and eHDECAY were discussed, in three reports of the Les Houches Workshops in 1999, 2009 and 2013, Refs. [27, 28, 29]. The logbook of all modifications and the most recent version 6.52 of the program can be found on the web page `http://tiger.web.psi.ch/hdecay/`. The most important updates are summarized in the next section.
2. The major updates and extensions of the program

2.1. Pre-Higgs discovery updates

Before Higgs boson discovery, most of the modifications of the original program have been made in the context of the MSSM. Until 1999 the following changes have been performed [27]:

- Added links to the *FeynHiggsFast* routine which provides the masses and couplings of the MSSM Higgs bosons up to two–loop order in the diagrammatic approach [30], and, in the framework of the *SUSY–HIT* program [12], to the *SUSPECT* routine for the renormalisation group evolution and for the proper electroweak symmetry breaking in the minimal supergravity model [31].

- Implemented Higgs boson decays to a gravitino and neutralino or chargino in gauge mediated SUSY breaking models [32] and SUSY–QCD corrections to Higgs boson decays to $q\bar{q}$ pairs (in particular bottom quarks) [33].

In 2003, a major step was made by providing an interface to the SUSY Les Houches Accord (SLHA) [34] and implementing it properly. This required several transformations of the corresponding renormalization schemes to the ones used by *HDECAY*. This option can be switched on and off by appropriate flags in the input file *hdecay.in*. The output file in the SLHA format can also be used again as input file for other programs (and *HDECAY* itself).

Before and at the 2009 Les Houches workshop, the following modifications, again mainly in the MSSM context, were implemented (some started to be made in the early 2000).

- Improvements of the SUSY–QCD corrections in neutral MSSM Higgs decays to $b\bar{b}$ [33] and the resummation of the $\Delta_b$ effects [35, 36] up to NNLO [37]. The corresponding $\Delta_b$ terms have also been included in charged Higgs decays $H^\pm \rightarrow tb$.

- Inclusion of the renormalization-group improved two-loop contributions to the MSSM Higgs self-interactions extending the results of Ref. [23] for the stop and sbottom contributions to arbitrary mixing parameters and mass splitting; see also Ref. [38].
– Inclusion of the full mass dependence of the NLO QCD corrections to the quark and squark loop contributions to photonic Higgs decays [39, 40]. This was also done in the SM Higgs case. (The decay widths can also be used to determine the production cross sections of Higgs bosons at photon colliders at NLO QCD.)

– In the context of the SM, inclusion of electroweak corrections to the SM Higgs boson decays $H \rightarrow W^{(*)}W^{(*)}/Z^{(*)}Z^{(*)} \rightarrow 4f$ in approximate form (including the $WW$ and $ZZ$ thresholds) which reproduces the full results of Refs. [41] within 1%. In this context double off-shell effects have also been extended to the Higgs-mass region above the $WW, ZZ$ thresholds.

– The full electroweak corrections to the gluonic SM Higgs decays have been implemented in terms of a grid that is used for interpolation [42]. This grid extends up to a Higgs mass of 1 TeV.

After 2009 a few further developments in the context of the MSSM have been implemented in HDECAY before the discovery of the Higgs particle:

– The sizeable SUSY–QCD corrections to MSSM Higgs boson decays into squark pairs [43] have been included supplemented by the improvements concerning the resummation of large contributions [44]. Within the same modification process the treatment of all squark masses and mixings has been moved to NLO using the approach and the scheme of Ref. [44].

– The MSSM strange Yukawa couplings have been extended to the inclusion of potentially large $\Delta_s$ effects and their resummation at the 1-loop level analogous to the $\Delta_b$ effects for the bottom Yukawa couplings [35, 36].

– Inclusion of running bottom mass and $\Delta_b$ effects in the top-quark decays $t \rightarrow H^{\pm}b$. For the MSSM (and later for the 2HDM) a new output file `br.top` is generated that provides the total top width and the corresponding branching ratios for $t \rightarrow Wb, H^{\pm}b$. 

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2.2. Post-Higgs discovery modifications: summary

In the subsequent years and during LHC operation, a large amount of work was devoted to improve the program. Many of the changes were made in order to meet the experimental needs and the recommendations of the LHC Higgs cross section working group. By 2013, i.e. a year after the Higgs discovery, very important modifications and additions to the program were made. They are summarized below.

- Inclusion of the leading SUSY–QCD and electroweak corrections to all effective down-type fermion Yukawa couplings, i.e. for \( \mu, \tau, s \) in addition to the bottom quark according to Refs. [18, 45]. In the MSSM the sneutrino mass parameters of the first two generations are allowed to be different from the third generation.

- Inclusion of the two-loop QCD corrections to top decays [46].

- Inclusion of the full CKM mixing effects in charged Higgs and top decays. This required the appropriate extension of the \texttt{hdecay.in} input file.

- Inclusion of running mass effects and \( \Delta_{b/s} \) corrections to the Yukawa couplings in charged Higgs decays into \( b \) and \( s \) quarks, where \( \Delta_{b/s} \) denotes the leading SUSY-QCD (and SUSY-electroweak in case of \( \Delta_b \)) corrections to the effective bottom/strange Yukawa couplings. Very recently subleading \( A_b \) terms have been included at NNLO in \( \Delta_b \) accompanied by their proper resummation [36, 47]. The NNLO results have been extended to the \( \Delta_s \) terms of the MSSM strange Yukawa couplings [47].

- Addition of the charged Higgs decays \( H^+ \rightarrow t \bar{d}/t \bar{s}/c \bar{d} \) including off-shell top quark contributions.

- Inclusion of charm loop contributions in the Higgs decays \( \phi \rightarrow gg \) for the SM and MSSM.

- Inclusion of the full electroweak corrections to SM Higgs decays \( H \rightarrow f \bar{f} \) [48] thus removing the approximation used before.

- Inclusion of the full NLO mass dependence of SM Higgs decays into gluons in terms of grids that extend to a Higgs mass of 1 TeV [40].
- Inclusion of a flag that allows to switch off all electroweak corrections to SM Higgs decays. This is relevant for consistently using the best possible predictions of the branching ratios for studies beyond the SM.

- The scheme and scale choices of the quark-mass input parameters have been changed to be in line with the conventions of the LHCHXWG [49]. This required in particular that the input values of the file hdecay.in have moved to the $\overline{\text{MS}}$ masses $m_b(\overline{m}_b)$ for the bottom and $m_c(3 \text{ GeV})$ for the charm quark. The corresponding bottom pole mass is determined internally by iterating the N$^3$LO matching relation [50]

\[
\overline{m}_b(m_b^{\text{OS}}) = \frac{m_b^{\text{OS}}}{1 + 3 \frac{\alpha_s(m_b^{\text{OS}})}{\pi} + K_b^{(1)} \left( \frac{\alpha_s(m_b^{\text{OS}})}{\pi} \right)^2 + K_b^{(2)} \left( \frac{\alpha_s(m_b^{\text{OS}})}{\pi} \right)^3}
\]

at the scale of the bottom pole mass$^3$ $m_b^{\text{OS}}$ where $K_b^{(1)} \sim 12.3$ and $K_b^{(2)} \sim 130.9$. The charm pole mass is determined from the (renormalon-free) relation [51]

\[
m_c^{\text{OS}} = m_b^{\text{OS}} - 3.41 \text{ GeV} \pm 0.01 \text{ GeV}
\]

In addition the scale of the input $\overline{\text{MS}}$ mass of the strange quark has been moved to 2 GeV to avoid sizeable non-perturbative effects when using 1 GeV as the input scale as in former versions of HDECAY. Finally the input values of the $W$ and $Z$ masses and widths should be chosen as the real parts of the complex poles that are related to the previous definitions of the physical pole masses $m_V^{\text{OS}}$ and decay widths $\Gamma_V^{\text{OS}}$ by

\[
m_V - i\Gamma_V = \frac{m_V^{\text{OS}} - i\Gamma_V^{\text{OS}}}{\sqrt{1 + \left( \frac{\Gamma_V^{\text{OS}}}{m_V^{\text{OS}}} \right)^2}}
\]

- The inclusion of the important option in which the Higgs couplings to fermions and massive gauge bosons are rescaled by constant factors in a simplified effective Lagrangian approach. This also allows to address the possibilities of fermiophobic or fermiophilic Higgs states.

$^3$Note that this leads to a slightly different value of the bottom pole mass compared to the matching at the scale $\overline{m}_b(\overline{m}_b)$ that is, however, within the corresponding uncertainty band [49].
– The possibility that a fourth generation of SM–like quarks and leptons is present has been included [52]. A significant impact emerges on the loop induced Higgs decays such as decays into gluons and photons but major changes also occur in tree-level decays in which the radiative corrections due the new fermions are extremely important.

– Extension of HDECAY to the general two-Higgs Doublet model (2HDM) [53]. This required the extension of the hdecay.in input file and the inclusion of several new decay modes that are not possible within the MSSM. The input file allows to work with two different sets of input variables for the 2HDM.

– Finally, the hMSSM option in the supersymmetric case has been implemented. In this case, the mass of the lightest CP–even MSSM Higgs boson \( h \), \( M_h = 125 \text{ GeV} \), fixes the dominant radiative corrections that enter the MSSM Higgs boson masses and couplings, leading to a Higgs sector that can be described, to a good approximation, by only two free parameters as it was the case at tree-level.

The last four major upgrades are discussed in separate subsections below.

2.3. Rescaled Higgs couplings

In 2013, the program HDECAY has been substantially modified (version 6.40) in order to cope with the possibility of modified Higgs couplings to fermions and massive gauge bosons. This was required by the LHC collaborations which started to measure precisely the Higgs production cross sections and the decay branching ratios, allowing to derive strong constraints on these couplings. To compare the experimental measurements with the theory predictions in the SM, it was convenient to allow for the variation of the different Higgs couplings to the other particles in a systematic way.

The inclusion of rescaled Higgs couplings to SM particles has been done according to the simplified effective interaction Lagrangian

\[
\mathcal{L}_{\text{int}} \supset - \left\{ \sum_{\psi} \frac{c_\psi m_{\psi} \bar{\psi} \psi}{v} + 2 c_W m_W^2 W^+ W^- + c_Z m_Z^2 Z^+ Z^- \right\} \frac{H}{v} \\
+ \left\{ \frac{\alpha_s}{8\pi} c_{gg} G^a_{\mu\nu} G^a_{\mu\nu} + \frac{\alpha}{8\pi} c_{\gamma\gamma} F^\mu_{\mu\nu} F^\nu_{\mu\nu} + \sqrt{\frac{\alpha}{4\pi}} c_{Z\gamma} F^\mu_{\mu\nu} z_{\mu\nu} \right\} \frac{H}{v} \tag{4}
\]
where $G^{\mu\nu}$, $F^{\mu\nu}$ and $Z^{\mu\nu}$ are the field strength tensors of the gluon, photon and $Z$-boson fields. The couplings $\alpha_s$, $\alpha$ and $\alpha_2$ are the strong, electromagnetic (in the Thompson limit) and SU(2) isospin ($g^2 = 4\pi\alpha_2$) couplings, respectively, $v$ is the Higgs vacuum expectation value and $H$ the Higgs boson field. The novel point-like couplings of the Higgs boson to gluons, photons and $Z$ bosons affect the Higgs decays $H \rightarrow gg/\gamma\gamma/Z\gamma$. Electroweak corrections are only kept in the SM part of the individual decay amplitudes, i.e. the parts for $c_\psi = c_W = c_Z = 1$ and $c_{gg} = c_{\gamma\gamma} = c_{Z\gamma} = 0$, while QCD corrections have been included in all parts of the decays widths, since the dominant parts factorize. This approach deviates from the general addition of dimension-six operators as pursued in Ref. [54] where additional tensor structures have been added at the dimension-six level.

The above rescaling of the Higgs couplings modifies e.g. the Higgs decay widths into quarks as

$$\Gamma(H \rightarrow q\bar{q}) = \frac{3G_FM_H}{4\sqrt{2\pi}}\overline{m}_q^2(M_H)c_b\{c_b + \delta_{elw}\}\left\{1 + \delta_{QCD} + \frac{c_t}{c_b}\delta_t\right\}$$

where $\delta_{elw}$ denotes the electroweak corrections [48], $\delta_{QCD}$ the pure QCD corrections [55, 56, 57], $\delta_t$ the top-quark induced QCD correction [58], and $\overline{m}_q$ is the running MS quark mass at the scale of the Higgs mass.

The gluonic Higgs decay, with the novel tensor structure involving the point-like coupling factor $c_{gg}$, is given by

$$\Gamma(H \rightarrow gg) = \frac{G_F\alpha_s^2M_H^3}{36\sqrt{2}\pi^3} \left[\sum_{Q=t,b,c} c_Q A_Q(\tau_Q) \right]^2 c_{eff}^2 \kappa_{soft}$$

$$+ \delta_{elw} \left(\sum_{Q,Q'=t,b,c} c_Q A_Q(\tau_Q) A_{Q'}^*(\tau_{Q'})\right) c_{eff}^2 \kappa_{soft}$$

$$+ 2\text{Re}\left(\sum_{Q=t,b,c} c_Q A_Q(\tau_Q) \frac{3}{2} c_{gg}\right) c_{eff} \kappa_{soft} + \left|\frac{3}{2} c_{gg}\right|^2 \kappa_{soft}$$

$$+ \sum_{Q,Q'=t,b} c_Q A_Q^*(\tau_Q) c_{Q'} A_Q(\tau_{Q'}) \kappa^{NLO}(\tau_Q, \tau_{Q'})\right\},$$

where $\tau_Q = 4m_Q^2/M_H^2$ and $\delta_{elw}$ denotes the electroweak corrections [42, 59]. The loop function $A_Q(\tau_Q)$ is normalized to unity for large quark masses and can be found in Ref. [40]. The contributions $c_{eff}$ and $\kappa_{soft}$ denote the QCD
corrections originating from the effective Lagrangian in the heavy top quark limit,
\[ \mathcal{L}_{\text{eff}} = c_{\text{eff}} \frac{\alpha_s}{12\pi} G^{\mu\nu} G_{\mu\nu}^a \frac{H}{v} \]  
(7)
and the residual corrections due to diagrams involving gluon exchange and light-quark contributions, respectively. They are included up to the next-to-next-to-next-to-leading order (NNNLO) [60, 61]. At NLO, they are given by [60],
\[ c_{\text{eff}} = 1 + \frac{11}{4} \frac{\alpha_s}{\pi}, \quad \kappa_{\text{soft}} = 1 + \left( \frac{73}{4} - \frac{7}{6} N_F \right) \frac{\alpha_s}{\pi} \]  
(8)
with \( N_F = 5 \) light quark flavours. Finally \( \kappa^{\text{NLO}} \) represents the finite top and bottom mass effects at NLO beyond the limit of heavy quarks, i.e. beyond the terms contained in \( c_{\text{eff}} \) and \( \kappa_{\text{soft}} \) [40].

All other Higgs decay modes are treated analogously in the case of rescaled Higgs couplings.

2.4. The fourth generation fermion option

In the four-generation fermion Standard Model (SM4), available since version 4.0 of the \textsc{HDecay} code, additional corrections to tree-level Higgs decays into fermions and massive gauge bosons arise from 4th-generation fermion loops. In the case of the \( H \rightarrow WW, ZZ \) decays, these corrections appear in the \( HWW/HZZ \) vertices, the \( W/Z \) self-energies, and the renormalization constants. Since the 4th-generation fermions are expected to be very heavy, their Yukawa couplings are large and they dominate the total corrections. Numerically the NLO corrections amount to about \(-50\%\) to \(-90\%\) in many cases and depend only weakly on the Higgs mass [52]. In the large fermion mass limit, the leading contribution can be absorbed into effective \( HWW/HZZ \) interactions via the Lagrangian
\[ \mathcal{L}_{HVV} = \sqrt{2} G_F H \left[ 2 M_W^2 W^+ W^- (1 + \delta_{W}^{\text{tot}}) + M_Z^2 Z^+ Z^- (1 + \delta_{Z}^{\text{tot}}) \right], \]  
(9)
where \( W, Z, H \) denote the fields for the respective states. The higher-order corrections are contained in the factors \( \delta_{V}^{\text{tot}} \) which, up to two-loop order, read
\[ \delta_{V}^{\text{tot}(1)} = \delta_{u}^{(1)} + \delta_{V}^{(1)}, \quad \delta_{V}^{\text{tot}(2)} = \delta_{u}^{(2)} + \delta_{V}^{(2)} + \delta_{u}^{(1)} \delta_{V}^{(1)}. \]  
(10)
The one-loop expressions for a single SU(2) doublet of heavy fermions with masses \( m_A, m_B \) read [62, 63]
\[ \delta_{u}^{(1)} = N_c X_A \left[ \frac{7}{6} (1 + x) + \frac{x}{1 - x} \ln x \right], \quad \delta_{V}^{(1)} = -2 N_c X_A (1 + x), \]  
(11)
where \( x = \frac{m_D^2}{m_A^2} \), \( X_A = \frac{G_F m_A^2}{8\sqrt{2}\pi^2} \), and \( N_c = 3 \) or \( 1 \) for quarks or leptons, respectively. The results for the two-loop corrections \( \delta_V^{\text{tot}(2)} \) can be found in [64] for the QCD corrections of \( \mathcal{O}(\alpha_s G_F m_T^2) \) and in [63] for the EW ones of \( \mathcal{O}(G_F^2 m_T^4) \). The corrected partial decay width is then given by

\[
\Gamma_{\text{NLO}} \approx \Gamma_{\text{LO}} \left[ 1 + \delta_V^{(1)} + \delta_V^{(2)} \right] = \Gamma_{\text{LO}} \left[ 1 + 2\delta_V^{\text{tot}(1)} + (\delta_V^{\text{tot}(1)})^2 + 2\delta_V^{\text{tot}(2)} \right]. \tag{12}
\]

In the case of Higgs decays into SM fermions, the decay widths \( \Gamma(H \to f \bar{f}) \) in the HDECAY code include, besides the SM corrections, the 4th generation approximate NLO and NNLO EW corrections in the heavy-fermion limit according to [63] and mixed NNLO EW/QCD corrections according to [64]. They originate from the wave-function renormalization of the Higgs boson and are thus universal for all fermion species. The leading one-loop part is given by \( \delta_V^{(1)} \) above. Numerically the EW one-loop correction to the partial widths amounts to a few tens of percent, while the two-loop EW and QCD correction contributes an additional few percent. These corrections are assumed to factorize since the approximate expressions emerge as corrections to the effective Lagrangian after integrating out the heavy fermion species. Thus, HDECAY multiplies the relative SM4 corrections with the full corrected SM3 (usual SM with three generations) result including QCD and EW corrections. The scale of the strong coupling \( \alpha_s \) is set to the average mass of the heavy 4th generation quarks according to the appropriate matching scale of the effective Lagrangian.

Turning to the loop induced decay modes \( H \to gg, \gamma \gamma, Z\gamma \), HDECAY includes the NNNLO QCD corrections of the SM in the limit of a heavy top quark [40, 60, 61], applied to the results including the heavy-quark loops. For \( H \to gg \), while at NNLO the exact QCD corrections in SM4 [65] are included in this limit, at NNNLO the relative SM3 corrections are added to the relative NNLO corrections and multiplied by the LO result including the additional quark loops. In addition the full NLO EW corrections [66] have been included in factorized form, since the dominant part of the QCD corrections emerges from the gluonic contributions on top of the corrections to the effective Lagrangian in the limit of heavy quarks.

HDECAY includes the full NLO QCD corrections to the decay mode \( H \to \gamma \gamma \) supplemented by the additional contributions of the 4th-generation quarks and charged leptons according to [39, 40]. Extending the techniques used for \( H \to gg \) in [66], the exact amplitude for \( H \to \gamma \gamma \) was included up to NLO (two-loop level) as in Ref. [52]. It required particular attention as in many
scenarios the correction is negative and larger than unity. This is due to the fact that at LO the cancellation between the $W$ and the fermion loops is stronger in SM4 than in SM3 so that the LO result is suppressed more. Furthermore, the NLO corrections are strongly enhanced for ultra-heavy fermions. In such a case, a proper estimate must also include the next term in the expansion [66] which is included in the grid implemented in HDECAY for $H \rightarrow \gamma\gamma$.

Finally, the decay mode $H \rightarrow \gamma Z$ is treated at LO only, since the NLO QCD corrections within the SM3 are known to be small [67] and can thus safely be neglected. The EW corrections in SM3 as well as in SM4 are unknown which implies a sizeable theoretical uncertainty within SM4, since large cancellations between the $W$ and fermion loops emerge at LO.

2.5. The two-Higgs doublet model extension

HDECAY version 6.0 has been extended to include the computation of the Higgs boson decay widths in the framework of the 2HDM [53]. The most general (CP-conserving) version of the 2HDM with a softly-broken $Z_2$-symmetry, i.e. type I–IV models, has been implemented. For the input parameters, to be specified in the input file hdecay.in, the user can choose between two different options given by

- the ratio of the vacuum expectation values: $\tan \beta$
- the mass parameter squared: $M_{12}^2$ (GeV$^2$)
- the quartic couplings of the Higgs potential: $\lambda_1, \ldots, \lambda_5$

or by a more physical basis

- the ratio of the vacuum expectation values: $\tan \beta$
- the mass parameter squared: $M_{12}^2$ (GeV$^2$)
- the CP–even Higgs mixing angle: $\alpha$
- the mass values of the five Higgs bosons: $M_h, M_H, M_A, M_{H^\pm}$ (GeV)

Furthermore, one can choose between the four 2HDM types by setting the corresponding flag accordingly. From these input parameters HDECAY calculates the couplings which are needed in the computation of the decay widths. With the appropriate coupling replacements according to the various 2HDMs, the decay widths are the same as for the MSSM Higgs boson decays, which are already included in the program. Only the SUSY particle contributions in the loop-mediated decays and the decays into SUSY particles as well as higher order corrections due to SUSY particle loops have been turned off for the 2HDM case.
The decay widths of the 2HDM Higgs bosons are usually calculated at LO in the 2HDM parameters. Unlike the case of the SM with a light Higgs or the MSSM, there is no automatic protection in the 2HDM against arbitrarily large quartic couplings, which may lead to a violation of perturbativity of the couplings and tree-level unitarity. This should be kept in mind when calculating decay widths involving Higgs self–couplings. The program also tests for vacuum stability, perturbative unitarity and the compatibility with the $S$ and $T$ parameters and gives out a warning if these are not fulfilled.

Higher-order SM EW corrections do not factorize from the LO result and cannot be readily included for the 2HDM. The higher-order EW corrections to all relevant 2HDM Higgs boson decays have only become available recently [68], and not been implemented in HDECAY yet. This introduces unavoidable uncertainties, which can be estimated from the size of the known EW SM corrections to be up to 5–10% for several partial decay widths. Differences of this magnitude compared to the most precise values for the SM are therefore expected even in the decoupling/alignment limit. Note, however, that the corrections can be much larger for Higgs-to-Higgs decays, where they can be parametrically enhanced.

A consistent comparison of 2HDM predictions in the decoupling/alignment limit to SM values furthermore requires that the limit is taken properly so that no residual 2HDM effects are present, e.g. from $H^\pm$ loop contributions to $h \to \gamma\gamma/Z\gamma$. Using the physical Higgs mass basis as an example, SM-like decays for the lightest 2HDM $h$ boson can be achieved by choosing $M_h \sim 125$ GeV, $\sin(\beta-\alpha)=1$, $M_{H,A,H^\pm}\gg v$, and $M_{12}^2$ such that $g_{hH^+H^-} \approx 0$.

Unlike EW corrections, many of the QCD corrections (which typically are numerically significant) do factorize, and can therefore be taken over directly from the corresponding SM or MSSM calculations. The widths for SM-type Higgs boson decays to quark and vector bosons pairs are obtained at LO from their SM equivalents by rescaling the vertices with the corresponding 2HDM factors. The loop-mediated decay to gluons also proceeds as in the SM, with the appropriate rescaling of the Yukawa couplings. For the remaining decays it is necessary in addition to take 2HDM-specific contributions (which cannot be assumed to be small) into account.

In HDECAY, the implemented decay widths and higher order corrections are specified in the following.

- Decays into quark pairs: The QCD corrections factorize and can be taken over from the SM case. For the neutral Higgs decays the fully
massive NLO corrections near threshold [55] and the massless $\mathcal{O}(\alpha_s^4)$
corrections far above threshold [56, 57, 58] are included in HDECAY. They are
calculated in terms of running quark masses and strong coupling
to resum large logarithms. The QCD corrections to $H^\pm$ decays have
been taken from [69]. The EW corrections cannot be adapted from
the SM case and are ignored. For the decays of the heavier neutral
Higgs bosons into a top quark pair, in HDECAY off-shell decays below
threshold have been implemented as well as for the decays of a charged
Higgs boson into a top-bottom quark pair [22].

- Decays into gluons: The QCD corrections to the neutral Higgs boson
decays into gluons, a loop-induced process already at leading order, can
be taken over from the SM, respectively, the MSSM. They have been
included up to N$^3$LO in HDECAY in the limit of heavy top quarks. While
for the SM at NLO the full quark mass dependence [40] is available, for
the 2HDM the corrections have been taken into account in the limit of
heavy-quark loop particle masses [40, 60, 61, 70].

- Decays into $\gamma\gamma, Z\gamma$: The decay to a photon pair is loop-mediated, with
the two most important SM contributions being due to the top quark
and $W$ boson loops. In the 2HDM, there is also a $H^+$ contribution
which becomes numerically significant in some cases. The bottom loop
becomes relevant in scenarios with enhanced bottom Yukawa couplings.
In the pseudoscalar case only heavy charged fermion loops contribute.
The neutral Higgs boson decays into a photon pair have been imple-
mented at NLO QCD including the full mass dependence for the quarks
[39, 40, 71]. The loop induced decays of scalar Higgs bosons into $Z\gamma$
are mediated by $W$, charged Higgs and heavy charged fermion loops,
while the pseudoscalar decays proceed only through charged fermion
loops. The QCD corrections to the quark loops are numerically small
[67] and have not been taken into account in HDECAY.

- Decays into massive gauge bosons: The decay widths of the scalar Higgs
bosons into massive gauge bosons $\phi \rightarrow V^{(*)}V^{(*)}$ are the same as the
SM decay width after replacing the SM Higgs coupling to gauge bosons
with the corresponding 2HDM Higgs coupling. The option of double
off-shell decays [21] has been included in HDECAY. The pseudoscalar
Higgs boson does not decay into massive gauge bosons at tree level.
Decays into Higgs boson pairs: The heavier Higgs particles can decay into a pair of lighter Higgs bosons. This is a feature of the 2HDM which does not exist in the SM. Due to more freedom in the mass hierarchies compared to the MSSM case, the following Higgs-to-Higgs decays are possible and have been taken into account in \texttt{HDECAY},

\[
h \to AA^{(*)}, \quad H \to hh^{(*)}, \quad H \to AA^{(*)}. \tag{13}
\]

Moreover the decays into a charged Higgs boson pair are possible\(^4\),

\[
h \to H^+H^-, \quad H \to H^+H^- . \tag{14}
\]

All decays are calculated at leading order using the tree-level expressions of the 2HDM trilinear couplings. The contributions from final states with an off-shell scalar or pseudoscalar, which can be significant, have been included in \texttt{HDECAY} \cite{22}. It is important to note that these partial widths can be very large for parameter points that do not respect the requirements of perturbativity and tree-level unitarity.

Decays into gauge and Higgs bosons: The Higgs boson decays into a gauge and a Higgs boson \cite{9,18}, which have been implemented in \texttt{HDECAY} including the possibility of off-shell gauge bosons \cite{22}, are in particular

\[
\begin{align*}
h & \to A\Z^{(*)}, & h & \to H^\pm W^{\mp(*)}, \\
H & \to AZ^{(*)}, & H & \to H^\pm W^{\mp(*)}, \\
A & \to h\Z^{(*)}, & A & \to hZ^{(*)}, & A & \to H^\pm W^{\mp(*)}, \\
H^\pm & \to hW^{\pm(*)}, & H^\pm & \to HW^{\pm(*)}, & H^\pm & \to AW^{\pm(*)} .
\end{align*}
\tag{15}
\]

They have been implemented at leading order and include the contributions of off-shell \(W\) and \(Z\) bosons below threshold \cite{22}.

\subsection*{2.6. The \(h\)MSSM scenario}

As mentioned earlier, in the MSSM, only two parameters are needed to describe the Higgs sector at tree-level. These are in general taken to be \(M_A\) and \(\tan\beta\). Nevertheless, when the radiative corrections \cite{73} are included in

\(^4\)Note that in type II and IV (flipped), certain decays are already kinematically closed due to the lower bound of \(M_{H^\pm} > 580\) GeV on the charged Higgs boson mass \cite{72}.
the Higgs sector, in particular the dominant loop contributions from the top and stop quarks that have strong couplings to the Higgs bosons [74], many supersymmetric parameters will enter the game. This is for instance the case of the SUSY scale, taken to be the geometric average of the two stop masses $M_S = \sqrt{m_{t1}m_{t2}}$, the stop/sbottom trilinear couplings $A_{t/b}$ or the higgsino mass $\mu$ (other corrections, that involve the gaugino mass parameters $M_{1,2,3}$ for instance are rather small).

In particular, the radiative corrections in the CP–even neutral Higgs sector are extremely important and shift the value of the lightest $h$ boson mass from the tree–level value $M_h \leq M_Z \cos 2\beta \leq M_Z$ to the value $M_h = 125$ GeV that has been measured experimentally. In the current-eigenstate basis of the Higgs fields $\Phi_1, \Phi_2$, the CP–even Higgs mass matrix including corrections can be written as:

$$M^2_S = M^2_Z \begin{pmatrix} c^2_\beta & -s_\beta c_\beta \\ -s_\beta c_\beta & s^2_\beta \end{pmatrix} + M^2_A \begin{pmatrix} s^2_\beta & -s_\beta c_\beta \\ -s_\beta c_\beta & c^2_\beta \end{pmatrix} + \begin{pmatrix} \Delta M^2_{11} & \Delta M^2_{12} \\ \Delta M^2_{12} & \Delta M^2_{22} \end{pmatrix},$$

where we have used the short–hand notation $c_\beta \equiv \cos \beta, s_\beta \equiv \sin \beta$ and the radiative corrections are captured by a general $2 \times 2$ matrix $\Delta M_{ij}$. The neutral CP–even Higgs boson masses and the mixing angle $\alpha$ that diagonalises the $h, H$ states can then easily be derived, $H = \Phi^0_1 \cos \alpha + \Phi^0_2 \sin \alpha$ and $h = -\Phi^0_1 \sin \alpha + \Phi^0_2 \cos \alpha$, where $\Phi^0_{1,2}$ denote the neutral CP–even components of the physical Higgs fields in the current-eigenstate basis. In the $2 \times 2$ matrix $\Delta M^2$, only the $\Delta M^2_{22}$ entry is in fact relevant in most cases (in particular if $\mu$ is small). It involves the by far dominant stop–top sector correction [74],

$$\Delta M^2_{22} \approx \Delta M^2_{h1\text{loop}} \sim \frac{3m_t^4}{2\pi^2 v^2} \left[ \log \frac{M^2_S}{m^2_t} + \frac{X_t^2}{M^2_S} - \frac{X_t^4}{12M^4_S} \right],$$

where $M_S$ is the SUSY scale and $X_t = A_t - \mu/\tan \beta$ the stop mixing parameter. Hence, one can write $\Delta M^2_{22} \gg \Delta M^2_{11}, \Delta M^2_{12}$ in general. It has been advocated [75, 76] that in this case, one can simply trade $\Delta M^2_{22}$ for the known $M_h$ value using

$$\Delta M^2_{22} = \frac{M^2_h(M^2_A + M^2_Z - M^2_h) - M^2_A M^2_Z c^2_{2\beta}}{M^2_A c^2_\beta + M^2_Z s^2_\beta - M^2_h}.$$  

17
One can then simply write $M_H$ and $\alpha$ in terms of $M_A$, $\tan \beta$ and $M_h$:

\[
M_H^2 = \frac{(M_A^2 + M_Z^2 - M_h^2)(M_Z^2 c_{\beta}^2 + M_A^2 s_{\beta}^2) - M_A^2 M_Z^2 c_{2\beta}}{M_Z^2 c_{\beta}^2 + M_A^2 s_{\beta}^2 - M_h^2} \\
\alpha = -\arctan \left( \frac{(M_Z^2 + M_A^2) c_{2\beta}}{M_Z^2 c_{\beta}^2 + M_A^2 s_{\beta}^2 - M_h^2} \right).
\] (19)

In the case of the $H^\pm$ masses, the radiative corrections are small at large enough $M_A$ and one has to a good approximation \[77\]

\[
M_{H^\pm} \simeq \sqrt{M_A^2 + M_W^2}.
\] (20)

This is the $h$MSSM approach which has been shown to provide a good approximation of the MSSM Higgs sector. In this $h$MSSM, the MSSM Higgs sector can be again described with only the two parameters $\tan \beta$ and $M_A$ as the loop corrections are fixed by the value of $M_h$. Another advantage of this approach is that it allows to describe the low $\tan \beta$ region of the MSSM which was overlooked as for SUSY scales of order 1 TeV, values $\tan \beta < 3$ were excluded because they lead to an $h$ mass that is smaller than 125 GeV. The price to pay is that for such low $\tan \beta$ values, one has to assume $M_S \gg 1$ TeV and, hence, that the model is fine-tuned. Moreover, care has to be taken not to enter regimes for small values of $\tan \beta$ that cannot be accommodated with the MSSM as pointed out in Ref. \[76\].

The couplings of the CP–even $h$ and $H$ to fermions and vector bosons are given in terms of the angle $\alpha$ which, including the radiative correction, is fixed by the $h$MSSM relations above. Additional direct corrections as $\Delta_b$ should in principle enter the Higgs couplings but because $M_S$ is taken to be very large, they are assumed to have a small impact in the $h$MSSM and are ignored. This, however, strongly depends on the size of the $\mu$ parameter and should be taken with caution for large values of $\tan \beta$. Another important set of couplings are the Higgs self-couplings and in the $h$MSSM, they are again given in terms of $\beta$ and $\alpha$, with the latter fixed by $\tan \beta$, $M_A$ and $M_h$ as in Eq. (19), but contain additional genuine radiative corrections that can be derived from the input parameters, too, since they are related to $\Delta M_{22}^2$.

The calculation of the Higgs branching ratios within the $h$MSSM are performed by \texttt{HDECAY} starting with version 6.40. The program takes $M_h$ as input and obtains $M_H$ and $\alpha$ from the $h$MSSM prescriptions. For the decays, the $h$MSSM mode of \texttt{HDECAY} implements: $N^4$LO-QCD corrections to the decays to quark pairs; LO results for the decays to lepton pairs and
for the decays involving massive gauge bosons, both on-shell and off-shell; a LO calculation of the decays to Higgs-boson pairs, both on-shell and off-shell, using effective $h$MSSM couplings such as the $Hhh$ coupling in particular. The triple Higgs couplings are an important issue [38] which needs some further studies in the $h$MSSM and some preliminary results recently appeared in [78].
3. The input file

In the following we list the input parameters of the hdecay.in input file along with some explanations\(^5\).

- **SLHAIN**: =0: READ FROM hdecay.in  
  =1: READ SUSY LES HOUCHES ACCORD INPUT (slha.in)
- **SLHAOUT**: =0: WRITE BR TABLES  
  =1: WRITE SUSY LES HOUCHES ACCORD OUTPUT (slha.out)
- **COUPVAR**: =0: NO VARIATION OF HIGGS COUPLINGS  
  =1: VARIATION OF HIGGS COUPLINGS (ONLY FOR SM)
- **HIGGS**: =0: CALCULATE BRANCHING RATIOS OF SM HIGGS BOSON  
  =1: CALCULATE BRANCHING RATIOS OF MSSM h BOSON  
  =2: CALCULATE BRANCHING RATIOS OF MSSM H BOSON  
  =3: CALCULATE BRANCHING RATIOS OF MSSM A BOSON  
  =4: CALCULATE BRANCHING RATIOS OF MSSM H+ BOSON  
  =5: CALCULATE BRANCHING RATIOS OF ALL MSSM HIGGS BOSONS
- **OMIT ELW**: =0: INCLUDE FULL ELECTROWEAK CORRECTIONS (SM)  
  =1: OMIT ALL ELECTROWEAK CORRECTIONS (SM)
- **SM4**: =0: CALCULATE USUAL BRANCHING RATIOS  
  =1: HIGGS WITH 4TH GENERATION (SETS HIGGS, FERMPHOB = 0)
- **FERMPHOB**: =0: CALCULATE USUAL BRANCHING RATIOS  
  =1: FERMIOPHOBIC HIGGS (SETS HIGGS = 0)
- **2HDM**: =0: CALCULATE USUAL BRANCHING RATIOS  
  =1: 2HDM (SETS HIGGS = 5)
- **MODEL**: USE SPECIFIC SUBROUTINE FOR MSSM HIGGS MASSES AND COUPLINGS  

\(^5\)The choices of the flag MODEL refer to the following References: MODEL = 1 [23], MODEL = 2 [79], MODEL = 3 [24], MODEL = 4 [30], MODEL = 10 [75].
TGBET: TAN(BETA) FOR MSSM
MABEG: START VALUE OF M_A FOR MSSM AND M_H FOR SM
MAEND: END VALUE OF M_A FOR MSSM AND M_H FOR SM
NMA: NUMBER OF ITERATIONS FOR M_A
MHL: LIGHT SCALAR HIGGS MASS FOR hMSSM (MODEL = 10)
ALS(MZ): VALUE FOR ALPHA_S(M_Z)
MSBAR(2): MSBAR MASS OF STRANGE QUARK AT SCALE Q=2 GEV
MCBAR(3): CHARM MSBAR MASS AT SCALE Q=3 GEV
MBBAR(MB): BOTTOM MSBAR MASS AT SCALE Q=MBBAR
MT: TOP POLE MASS
MTAU: TAU MASS
MMUON: MUON MASS
ALPH: INVERSE QED COUPLING
GF: FERMI CONSTANT
GAMW: W WIDTH
GAMZ: Z WIDTH
MZ: Z MASS
MW: W MASS
VTB: CKM PARAMETER |V_TB|
VTS: CKM PARAMETER |V_TS|
VTD: CKM PARAMETER |V_TD|
VCB: CKM PARAMETER |V_CB|
VCS: CKM PARAMETER |V_CS|
VCD: CKM PARAMETER |V_CD|
VUB: CKM PARAMETER |V_UB|
VUS: CKM PARAMETER |V_US|
VUD: CKM PARAMETER |V_UD|
GG_ELW: SCENARIO OF THE ELW. CORRECTIONS TO H -> GG (4TH GENERATION)
MTP: TOP’ MASS (4TH GENERATION)
MBP: BOTTOM’ MASS (4TH GENERATION)
MNUP: NU’ MASS (4TH GENERATION)
MEP: E’ MASS (4TH GENERATION)

2HDM models

TYPE: 1 (type I), 2 (type II), 3 (lepton-specific), 4 (flipped)
PARAM:  1 (masses), 2 (lambda_i)
TGBET2HDM:  TAN(BETA)
ALPHA_H:  MIXING ANGLE IN THE CP-EVEN neutral HIGGS SECTOR
MHL:  MASS OF THE LIGHT CP-EVEN HIGGS BOSON
MHH:  MASS OF THE HEAVY CP-EVEN HIGGS BOSON
MHA:  MASS OF THE CP-ODD HIGGS BOSON
MH+-:  MASS OF THE CHARGED HIGGS BOSONS
LAMBDA1:  2HDM lambda parameter
LAMBDA2:  2HDM lambda parameter
LAMBDA3:  2HDM lambda parameter
LAMBDA4:  2HDM lambda parameter
LAMBDA5:  2HDM lambda parameter
M_12^2:  2HDM parameter M12 SQUARED
SUSYSSCALE:  SCALE FOR SUSY BREAKING PARAMETERS
1ST AND 2ND GENERATION:
MSL1:  SUSY BREAKING MASS PARAMETERS OF LEFT HANDED SLEPTONS
MER1:  SUSY BREAKING MASS PARAMETERS OF RIGHT HANDED SLEPTONS
MQL1:  SUSY BREAKING MASS PARAMETERS OF LEFT HANDED SUPS
MUR1:  SUSY BREAKING MASS PARAMETERS OF RIGHT HANDED SUPS
MDR1:  SUSY BREAKING MASS PARAMETERS OF RIGHT HANDED SDOWNS
3RD GENERATION:
MSL:  SUSY BREAKING MASS PARAMETERS OF LEFT HANDED STAUS
MER:  SUSY BREAKING MASS PARAMETERS OF RIGHT HANDED STAUS
MSQ:  SUSY BREAKING MASS PARAMETERS OF LEFT HANDED STOPS
MUR:  SUSY BREAKING MASS PARAMETERS OF RIGHT HANDED STOPS
MDR:  SUSY BREAKING MASS PARAMETERS OF RIGHT HANDED SBOTTOMS
AL:  STAUS TRILINEAR SOFT BREAKING TERMS
AU:  STOP TRILINEAR SOFT BREAKING TERMS
AD:  SBOTTOM TRILINEAR SOFT BREAKING TERMS
MU:  SUSY HIGGS MASS PARAMETER
M2:  GAUGINO MASS PARAMETER
MGLUINO:  GLUINO POLE MASS
ON-SHELL:  =0: INCLUDE OFF-SHELL DECAYS H,A --> T*T*, A --> Z*H,  
            =1: EXCLUDE THE OFF-SHELL DECAYS ABOVE
ON-SH-WZ:  =0: INCLUDE DOUBLE OFF-SHELL PAIR DECAYS PHI --> W*W*,Z*Z*
            =1: INCLUDE DOUBLE OFF-SHELL PAIR DECAYS PHI --> W*W*,Z*Z*
BELOW THRESHOLD, BUT ON-SHELL PAIR DECAYS ABOVE
\(-1\): INCLUDE ONLY SINGLE OFF-SHELL DECAYS PHI --> W*W,Z*Z
BELOW THRESHOLD, BUT ON-SHELL PAIR DECAYS ABOVE

\textbf{IPOLE:} \(=0\) COMPUTES RUNNING HIGGS MASSES (FASTER)
\(=1\) COMPUTES POLE HIGGS MASSES

\textbf{OFF-SUSY:} \(=0\): INCLUDE DECAYS (AND LOOPS) INTO SUPERSYMMETRIC PARTICLES
\(=1\): EXCLUDE DECAYS (AND LOOPS) INTO SUPERSYMMETRIC PARTICLES

\textbf{INDIDEC:} \(=0\): PRINT OUT SUMS OF CHARGINO/NEUTRALINO/SFERMION DECAYS
\(=1\): PRINT OUT INDIVIDUAL CHARGINO/NEUTRALINO/SFERMION DECAYS

\textbf{NF-GG:} NUMBER OF LIGHT FLAVORS INCLUDED IN THE GLUONIC DECAYS
PHI --> GG* --> GQQ (3, 4 OR 5)

\textbf{IGOLD:} \(=0\): EXCLUDE DECAYS INTO GRAVITINO + GAUGINO
\(=1\): INCLUDE DECAYS INTO GRAVITINO + GAUGINO

\textbf{MPLANCK:} PLANCK MASS FOR DECAYS INTO GRAVITINO + GAUGINO
\textbf{MGOLD:} GRAVITINO MASS FOR DECAYS INTO GRAVITINO + GAUGINO

\textbf{RESCALING OF COUPLINGS}

\textbf{ELWK:} \(= 0\): Include elw. corrections only for SM part
\(= 1\): Include elw. corrections in all rescalings of couplings

\textbf{CW:} RESCALING FACTOR OF HWW COUPLING
\textbf{CZ:} RESCALING FACTOR OF HZZ COUPLING
\textbf{Ctau:} RESCALING FACTOR OF HTAUTAU COUPLING
\textbf{Cmu:} RESCALING FACTOR OF HMUMU COUPLING
\textbf{Ct:} RESCALING FACTOR OF HTT COUPLING
\textbf{Cb:} RESCALING FACTOR OF HBB COUPLING
\textbf{Cc:} RESCALING FACTOR OF HCC COUPLING
\textbf{Cs:} RESCALING FACTOR OF HSS COUPLING
\textbf{Cgaga:} POINT-LIKE H-GAMMA-GAMMA COUPLING
\textbf{Cgg:} POINT-LIKE HGG COUPLING
\textbf{CZga:} POINT-LIKE H-Z-GAMMA COUPLING

4th generation fermions
In the next section we will give sample output files for the various models implemented in HDECAY. The input file that we use to generate the outputs is given here. We will then later only indicate the changes of those parameters in the input file that are relevant for the specific model. The input file hdecay.in reads

```
SLHAIN = 0
SLHAOUT = 0
COUPVAR = 0
HIGGS = 0
OMIT ELW = 0
SM4 = 0
FERMPHOB = 0
2HDM = 0
MODEL = 1
TGBET = 30.D0
MABEG = 125.D0
MAEND = 1000.D0
NMA = 1

******************** hMSSM (MODEL = 10) ********************
MHL = 125.D0

******************** hMSSM (MODEL = 10) ********************
ALS(MZ) = 0.1180D0
MSBAR(2) = 0.095D0
MCBAR(3) = 0.986D0
MBBAR(MB)= 4.180D0
MT = 173.2D0
MTAU = 1.77682D0
MMUON = 0.1056583715D0
1/ALPHA = 137.0359997D0
GF = 1.1663787D-5
GAMW = 2.08430D0
GAMZ = 2.49427D0
MZ = 91.15348D0
NW = 80.35797D0
VTB = 0.9991D0
```
VTS  = 0.0404D0
VTD  = 0.00867D0
VCB  = 0.0412D0
VCS  = 0.97344D0
VCD  = 0.22520D0
VUB  = 0.00351D0
VUS  = 0.22534D0
VUD  = 0.97427D0

********************* 4TH GENERATION *******************************

SCENARIO FOR ELW. CORRECTIONS TO H -> GG (EVERYTHING IN GEV):

GG_ELW = 1: MTP = 500 MBP = 450 MNUP = 375 MEP = 450
GG_ELW = 2: MBP = MNUP = MEP = 600 MTP = MBP+50*(1+LOG(M_H/115)/5)

GG_ELW = 1
MTP  = 500.D0
MBP  = 450.D0
MNUP = 375.D0
MEP  = 450.D0

******************* 2 Higgs Doublet Model ***************************

TYPE: 1 (I), 2 (II), 3 (lepton-specific), 4 (flipped)
PARAM: 1 (masses), 2 (lambda_i)

PARAM = 1
TYPE  = 2

TGBET2HDM= 1.29775D0
M_12^2  = 82857.8D0

*********************** PARAM=1:
ALPHA_H  = -0.684653D0
MHL     = 125.09D0
MHH     = 453.87D0
MHA     = 591.552D0
MH+-    = 613.93D0

*********************** PARAM=2:
LAMBDA1 = 0.989175D0
LAMBDA2 = 0.734211D0
LAMBDA3 = 6.42606D0
LAMBDA4 = -3.83528D0
LAMBDA5 = -2.94533D0

**************************************************************************
SUSYSCALE= 500.D0
MU = 200.D0
M2 = 200.D0
MGLUINO = 1500.D0
MSL1 = 1000.D0
MER1 = 1000.D0
MQL1 = 1000.D0
MUR1 = 1000.D0
MDR1 = 1000.D0
MSL = 1000.D0
MER = 1000.D0
MSQ = 1000.D0
MUR = 1000.D0
MDR = 1000.D0
AL = 1607.D0
AU = 1607.D0
AD = 1607.D0
ON-SHELL = 0
ON-SH-WZ = 0
IPOLE = 0
OFF-SUSY = 0
INDIDEC = 0
NF-GG = 5
IGOLD = 0
MPLANCK = 2.4D18
MGOLD = 1.D-13
***************************************************************************** VARIATION OF HIGGS COUPLINGS*****************************************************************************
ELWK = 0
CW = 1.D0
CZ = 1.D0
Ctau = 1.D0
Cmu = 1.D0
Ct = 1.D0
Cb = 1.D0
Cc = 1.D0
Cs = 1.D0
Cgaga = 0.D0
Cgg = 0.D0
CZga = 0.D0
***************************************************************************** 4TH GENERATION*****************************************************************************

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4. The output files

We will give exemplary output files for the SM, the 2HDM and the MSSM based on the input file given above with the respective changes for these models given below.

4.1. The Standard Model

The input file can be taken over without any changes. It leads to the following \texttt{br.sm1} and \texttt{br.sm2} output files given by

\begin{verbatim}
MHSM  BB  TAU  TAU  MU  MU  SS  CC  TT
125.000  0.5811  0.6259E-01  0.2172E-03  0.2239E-03  0.2886E-01  0.000
\end{verbatim}

and

\begin{verbatim}
MHSM  GG  GAM  GAM  Z  GAM  WW  ZZ  WIDTH
125.000  0.8164E-01  0.2265E-02  0.1529E-02  0.2152  0.2634E-01  0.4096E-02
\end{verbatim}

respectively, for the branching ratios of the SM Higgs with mass $M_{H_{SM}} = 125$ GeV into the bottom-quark, tau- and muon-pair, strange-, charm- and top-quark pair final states as well as into gluon, photon, $Z\gamma$ and massive gauge boson final states. The last entry in \texttt{br.sm2} is the total width in GeV.

4.2. The 2HDM

For the 2HDM example we chose a scenario compatible with all relevant theoretical and experimental constraints \cite{80} and also implies a strong first order phase transition as required by baryogenesis \cite{81}. It induces a mass spectrum where the lightest CP-even Higgs boson is the SM-like Higgs state. The input parameters are specified in the above input file, and only the following two parameters need to be changed to produce the 2HDM output files:

\begin{verbatim}
HIGGS  =  5
2HDM   =  1
\end{verbatim}
For the given scenario the input is the ‘physical’ one via the Higgs masses and the mixing angles and PARAM is set equal to 1. If PARAM is set equal to 2 the $\lambda_i$’s as given in the above input file lead to the same results. The output for the branching ratios is given in the files br.xy_2HDM with $x = l, h, a, c$ for the light and heavy CP-even $h$ and $H$ states, for the CP-odd Higgs $A$ and the charged boson $H^{\pm}$, respectively. The index $y$ counts the output files of each Higgs boson. The three output files br.11_2HDM, br.12_2HDM and br.13_2HDM for the SM-like Higgs with mass $m_h = 125.09$ GeV read:

<table>
<thead>
<tr>
<th>MHL</th>
<th>BB</th>
<th>TAU TAU</th>
<th>MU MU</th>
<th>SS</th>
<th>CC</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>125.090</td>
<td>0.6080</td>
<td>0.6542E-01</td>
<td>0.2316E-03</td>
<td>0.2294E-03</td>
<td>0.2653E-01</td>
<td>0.000</td>
</tr>
<tr>
<td>MHL</td>
<td>GG</td>
<td>GAM GAM</td>
<td>Z GAM</td>
<td>WW</td>
<td>ZZ</td>
<td></td>
</tr>
<tr>
<td>125.090</td>
<td>0.7041E-01</td>
<td>0.2126E-02</td>
<td>0.1458E-02</td>
<td>0.2005</td>
<td>0.2507E-01</td>
<td></td>
</tr>
<tr>
<td>MHL</td>
<td>AA</td>
<td>Z A</td>
<td>W+- H+- H+ H-</td>
<td>WIDTH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>125.090</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.4248E-02</td>
<td></td>
</tr>
</tbody>
</table>

As can be inferred from these files the $h$ boson behaves SM-like. For $H$ we obtain the three output files br.h1_2HDM, br.h2_2HDM and br.h3_2HDM:

<table>
<thead>
<tr>
<th>MHH</th>
<th>BB</th>
<th>TAU TAU</th>
<th>MU MU</th>
<th>SS</th>
<th>CC</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>453.870</td>
<td>0.1869E-02</td>
<td>0.2595E-03</td>
<td>0.9176E-06</td>
<td>0.6831E-06</td>
<td>0.3675E-04</td>
<td>0.9807</td>
</tr>
<tr>
<td>MHH</td>
<td>GG</td>
<td>GAM GAM</td>
<td>Z GAM</td>
<td>WW</td>
<td>ZZ</td>
<td></td>
</tr>
<tr>
<td>453.870</td>
<td>0.3781E-02</td>
<td>0.1094E-04</td>
<td>0.2425E-05</td>
<td>0.3372E-02</td>
<td>0.1595E-02</td>
<td></td>
</tr>
<tr>
<td>MHH</td>
<td>hh</td>
<td>AA</td>
<td>Z A</td>
<td>W+- H+- H+ H-</td>
<td>WIDTH</td>
<td></td>
</tr>
<tr>
<td>453.870</td>
<td>0.8341E-02</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>5.837</td>
</tr>
</tbody>
</table>

The heavy Higgs boson with mass above 350 GeV dominantly decays into a top-quark pair. It can also decay into a pair of lighter Higgs bosons. The branching ratio is rather small, however. The three output files br.a1_2HDM, br.a2_2HDM and br.a3_2HDM of the pseudoscalar Higgs boson are given by:

---

6Note that in the present version of HDECAY no SLHA output files are provided in the 2HDM case.
Also the pseudoscalar dominantly decays into a top-quark pair. However, the decay into the Z boson and the heavy Higgs boson $H$ contributes with 10% and is a prime example of a beyond-the-SM decay. For the charged Higgs boson the three generated output files $\text{br.c1.2HDM}$, $\text{br.c2.2HDM}$ and $\text{br.c3.2HDM}$ read:

<table>
<thead>
<tr>
<th>MHC</th>
<th>BB</th>
<th>TAU</th>
<th>TAU</th>
<th>MU</th>
<th>MU</th>
<th>SS</th>
<th>CC</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>591.552</td>
<td>0.7844E-03</td>
<td>0.1121E-03</td>
<td>0.3965E-06</td>
<td>0.2950E-06</td>
<td>0.1365E-04</td>
<td>0.8873</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| MHC | GG | GAM | GAM | Z | GAM | Z | h | Z | H |
|-----|----|-----|-----|---|-----|---|---|---|
| 591.552 | 0.2648E-02 | 0.7803E-05 | 0.2312E-05 | 0.2341E-02 | 0.1068 |

<table>
<thead>
<tr>
<th>MHC</th>
<th>W++</th>
<th>H++</th>
<th>WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>591.552</td>
<td>0.000</td>
<td>18.41</td>
<td></td>
</tr>
</tbody>
</table>

The first two files contain the branching ratios for the fermionic final states, and the last one the charged Higgs branching ratios into Higgs-gauge boson final states, which can become significant here in the $HW$ case. In addition the top-quark branching ratios and total width are given in the file $\text{br.top}$:

<table>
<thead>
<tr>
<th>MHC</th>
<th>W++</th>
<th>H++</th>
<th>WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>613.930</td>
<td>1.000</td>
<td>0.000</td>
<td>1.336</td>
</tr>
</tbody>
</table>

For this 2HDM scenario the top quark decays entirely into $Wb$ final states.
4.3. The MSSM

In order to generate the output file for the MSSM with a SM-like Higgs boson mass close to 125 GeV, we have to change in the input file given above

\[ \text{HIGGS} = 5 \]
\[ \text{MABEG} = 1000.0 \]

The scenario that corresponds to the MSSM is the \( m_h^{\text{mod+}} \) scenario defined in Ref. [82]. It induces a mass spectrum with the lightest CP-even Higgs boson having a mass close to 125 GeV, namely \( m_h = 122.644 \) GeV. The branching ratios, given in \text{br.11} and \text{br.12}, are

<table>
<thead>
<tr>
<th>MHL</th>
<th>BB</th>
<th>TAU</th>
<th>TAU</th>
<th>MU</th>
<th>MU</th>
<th>SS</th>
<th>CC</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>122.644</td>
<td>0.6419</td>
<td>0.6686E-01</td>
<td>0.2367E-03</td>
<td>0.2348E-03</td>
<td>0.2905E-01</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MHL</th>
<th>GG</th>
<th>GAM</th>
<th>GAM</th>
<th>Z</th>
<th>GAM</th>
<th>WW</th>
<th>ZZ</th>
<th>WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>122.644</td>
<td>0.7593E-01</td>
<td>0.2195E-02</td>
<td>0.1263E-02</td>
<td>0.1628</td>
<td>0.1953E-01</td>
<td>0.3993E-02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The file \text{br.1s} includes the branching ratios into the SUSY particle final states, which are all kinematically closed, however, so that we do not give \text{br.1s} separately here. Being in the decoupling limit with the chosen large pseudoscalar mass of 1 TeV, the branching ratios are close to those of a SM Higgs boson with same mass. For \( H \), the branching ratios into SM particle and Higgs boson final states listed in \text{br.h1}, \text{br.h2}, \text{br.h3} amount to:

<table>
<thead>
<tr>
<th>MHH</th>
<th>BB</th>
<th>TAU</th>
<th>TAU</th>
<th>MU</th>
<th>MU</th>
<th>SS</th>
<th>CC</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.02</td>
<td>0.4215</td>
<td>0.7915E-01</td>
<td>0.2799E-03</td>
<td>0.1627E-03</td>
<td>0.3208E-07</td>
<td>0.2062E-02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MHH</th>
<th>GG</th>
<th>GAM</th>
<th>GAM</th>
<th>Z</th>
<th>GAM</th>
<th>WW</th>
<th>ZZ</th>
<th>WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.02</td>
<td>0.5130E-04</td>
<td>0.3036E-06</td>
<td>0.2152E-07</td>
<td>0.9420E-05</td>
<td>0.4652E-05</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MHH</th>
<th>hh</th>
<th>AA</th>
<th>Z</th>
<th>A</th>
<th>W+</th>
<th>H+</th>
<th>H-</th>
<th>H+ H-</th>
<th>WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.02</td>
<td>0.4391E-04</td>
<td>0.8720E-23</td>
<td>0.4605E-19</td>
<td>0.000</td>
<td>0.000</td>
<td>23.78</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Due to the large value of \( \tan \beta = 30 \) the branching ratio into \( b\bar{b} \) dominates over the one into \( tt \). While the decay into the light Higgs boson pair is kinematically open, it is very small. The decay into \( AA \) is far off-shell and hence tiny\(^7\). The decays into SUSY particles are given in \textit{br.hs}:

\[
TB= 30.0000 \quad M2= 200.000 \quad MU= 200.000 \quad MSQ= 1000.00 \\
C1=148.714 \quad C2= 266.081 \quad N1= 88.414 \quad N2=152.084 \quad N3= 210.462 \quad N4= 265.541 \\
MST1= 856.771 \quad MST2= 1101.73 \quad MSUL= 975.811 \quad MSUR= 976.706 \\
MSB1= 973.103 \quad MSB2= 983.796 \quad MSDL= 979.235 \quad MSDR= 977.688 \\
TAU1= 997.129 \quad TAU2=1004.930 \quad NL= 997.925 \quad EL= 1001.15 \quad ER= 1000.92 \\
NL1= 997.93
\]

<table>
<thead>
<tr>
<th>MHH</th>
<th>CHARGINOS</th>
<th>NEUTRALS</th>
<th>SLEPTONS</th>
<th>SQUARKS</th>
<th>GRAVITINO+GAUGIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000.02</td>
<td>0.3027</td>
<td>0.1940</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The kinematically allowed decays into SUSY particles, on the other hand, are important with branching ratios into charginos and neutralinos of about 30\% and 20\%, respectively. Note that the SUSY particle branching ratios given in the output files sum up all the final states of the same SUSY particle type. In \textit{br.hs} also the masses of the SUSY particles are repeated in the output for convenience. The branching ratios of the pseudoscalar are summarized in the output files \textit{br.a1}, \textit{br.a2} and \textit{br.as}:

\[
MHA \quad BB \quad TAU \quad TAU \quad MU \quad MU \quad SS \quad CC \quad TT \\
1000.00 | 0.4216 | 0.7916E-01 | 0.2799E-03 | 0.1627E-03 | 0.3078E-07 | 0.2158E-02
\]

\[
MHA \quad GG \quad GAM \quad GAM \quad Z \quad GAM \quad Z \quad HL \quad WIDTH \\
1000.00 | 0.9014E-04 | 0.5085E-06 | 0.3919E-07 | 0.9280E-05 | 23.78
\]

and

\[
TB= 30.0000 \quad M2= 200.000 \quad MU= 200.000 \quad MSQ= 1000.00 \\
C1=148.714 \quad C2= 266.081 \quad N1= 88.414 \quad N2=152.084 \quad N3= 210.462 \quad N4= 265.541 \\
MST1= 856.771 \quad MST2= 1101.73 \quad MSUL= 975.811 \quad MSUR= 976.706
\]

\(^7\)Tiny negative value of the branching ratio may arise due to an artefact of the finite accuracy of the implemented expressions in the Fortan code and should be ignored, i.e. identified with zero.
Also the pseudoscalar has significant decay rates into charginos and neutralinos. The charged Higgs branching ratios finally, given in $\text{br.c1}$, $\text{br.c2}$, $\text{br.c3}$ and $\text{br.cs}$, are

<table>
<thead>
<tr>
<th>MHC</th>
<th>BC</th>
<th>TAU</th>
<th>NU</th>
<th>MU</th>
<th>NU</th>
<th>SU</th>
<th>CS</th>
<th>TB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1002.86</td>
<td>0.7082E-03</td>
<td>0.8290E-01</td>
<td>0.2931E-03</td>
<td>0.8482E-05</td>
<td>0.1583E-03</td>
<td>0.4019</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MHC</th>
<th>CD</th>
<th>BU</th>
<th>TS</th>
<th>TD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1002.86</td>
<td>0.6168E-08</td>
<td>0.5140E-05</td>
<td>0.3834E-05</td>
<td>0.1645E-06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MHC</th>
<th>hW</th>
<th>AW</th>
<th>WIDTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1002.86</td>
<td>0.9836E-05</td>
<td>0.1109E-10</td>
<td>22.77</td>
</tr>
</tbody>
</table>

The decay into $hW$, although kinematically allowed, is very small, the one into $AW$ is far off-shell and hence tiny. The decay branching ratio for the chargino-neutralino final states amounts to more than 50% and is dominating. Finally the branching ratios of the top quark as given in the file $\text{br.top}$ read
i.e. the top quark decays entirely into $Wb$ final states.

**Acknowledgments**

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**References**


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[76] E. Bagnaschi et al., Benchmark scenarios for low tan $\beta$ in the MSSM, Note LHCHXSWG-2015-002.


